

**LECTURE NOTES**  
**ON**  
**FLUID MECHANICS & FLUID POWER**



**3<sup>RD</sup> SEMESTER**

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# (Fluid Mechanics and Hydraulic Machine)

Chapters includes →

- (1) Properties of Fluid
- (2) Pressure at a point
- (3) Buoyancy and Floatation
- (4) Hydrostatic Forces on Surfaces
- (5) Fluid Kinematics
- (6) Fluid Dynamics
- (7) Concept of Laminar & Turbulent Flow
- (8) Impulse and Reaction Turbine
- (9) Centrifugal and Reciprocating Pump

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## Fluid Mechanics

It is the branch of science which deals with the behaviour of fluid under Rest and also motion. It is also a Branch of applied Mechanics. It is again divided into two types.

- (1) Statics  $\rightarrow$  Behaviour of fluid under static condition.
- (2) Kinematics  $\rightarrow$  deals with the study of velocity, acceleration
- (3) Dynamics  $\rightarrow$  deals with the study of Relationship of velocity, acceleration of fluid with different energy causing them.

## Concept of fluid

Fluid is generally the combination of both liquid and gaseous state. In liquid the spacing between the molecules are relatively large and in case of gases also it is large. Fluid can resist only compressive forces when stored in a container. Fluid can also deform continuously when we are applying shear force.

Fluid is substance which is capable of flowing under the application of shear force how small the shear force may be.

## Continuum Concept

A continuous and homogenous medium is called continuum. From this concept we analyze different properties of fluid.

## Properties of fluid

(1) mass density  $\rightarrow \rho = \frac{\text{mass}}{\text{volume}} \text{ (kg/m}^3\text{)}$

(2) weight density  $\rightarrow$  weight per unit volume.

$$w = \frac{W}{V} \text{ (kN/m}^3\text{)}$$

(3) Specific volume  $\rightarrow$  Volume per unit mass

$$v = \frac{V}{m} = \frac{1}{\rho}$$

(4) Specific gravity  $\rightarrow$  Specific gravity is the Ratio between Specific weight of liquid to the specific weight of standard fluid.

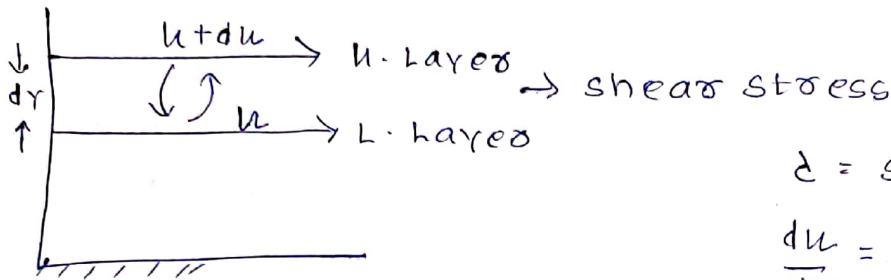


It has no dimension.

$$s.g = \frac{w_{\text{liquid}}}{w_{\text{water}}}$$

(5) **VISCOSITY**  $\rightarrow$  It is the most important properties of fluid which determines its resistance to shearing stress, it is mainly due to cohesion and molecular momentum exchange.

Let's consider two fluid layers



$$\tau \propto \frac{du}{dy}$$

$\tau$  = shear stress

$$\frac{du}{dy} = \frac{\text{rate change of velocity}}{\text{distance bet}^n \text{ them}}$$

$$\tau = \mu \times \frac{du}{dy}$$

$$\boxed{\mu = \frac{\tau}{\frac{du}{dy}}} = \text{co-efficient of dynamic viscosity / viscosity}$$

$\frac{du}{dy}$  = velocity gradient = Rate of shear deformation.

S.I unit is  $N \cdot s / m^2$  or

c.g.s also called poise. Dimension is  $[M^1 L^{-1} T^{-1}]$

(6) **Kinematic viscosity**  $\rightarrow$  It is defined as the Ratio between dynamic viscosity to the density of the fluid.

$$\boxed{\nu = \frac{\mu}{\rho}}$$

c.g.s unit is stoke  
S.I unit is  $m^2/sec$

### Newton's Law of viscosity

This law state that shear stress on a fluid element is directly proportional to the rate of shear strain.

$$\tau \propto \frac{du}{dy} \Rightarrow \tau = \mu \times \frac{du}{dy}$$

### Types of fluid

(1) **Newtonian fluid**  $\rightarrow$  These fluid which obey Newton's laws of viscosity. ex  $\rightarrow$  water, kerosene, air etc.

(2) **NON-Newtonian Fluid**  $\rightarrow$  The fluid which does not obey the Newton's law is called non-newtonian fluid. Solution of polymer, blood etc.

(3) **Ideal Fluid**  $\rightarrow$  An ideal fluid which is incompressible and having no viscosity.

(4) **Types of non-newtonian fluid**  $\rightarrow$

**Ideal plastic fluid**  $\rightarrow \tau = A + \mu \left( \frac{du}{dy} \right)$  ex  $\rightarrow$  Drilling muds, Sewage

**Thixotropic substance**  $\rightarrow \tau = A + \mu \left( \frac{du}{dy} \right)^n$  ex  $\rightarrow$  Printers Ink

**Pseudo plastic fluid**  $\rightarrow \tau = \mu x \left( \frac{du}{dy} \right)^n$  ( $n < 1$ ) ex  $\rightarrow$  Paper pulp

**Dilatant fluid**  $\rightarrow \tau = \mu x \left( \frac{du}{dy} \right)^n$  when ( $n > 1$ ) ex  $\rightarrow$  Butter

### Effect of temperature on viscosity

The viscosity of liquid decreases but that of gases increases with increase in temperature. In case of liquid we have a greater value of shear stress and cohesion which decrease with increase in temp while in cases of gases the cohesion is negligible so that it increases with increase in temperature.

### Concept of cohesion and Adhesion

Cohesion is the intermolecular attraction between the molecules of the same liquid. It enables the liquid to resist a small amount of tensile stress.

Adhesion means attraction between the molecules of a liquid and the molecules of another liquid. which enables a liquid to stick with another.

### Surface tension

It is due to the force of cohesion at the free surface as we know a liquid molecule is surrounded by various molecules which gives rise to a tensile force. example

(1) Rain drops

(2) Bird can drink water from

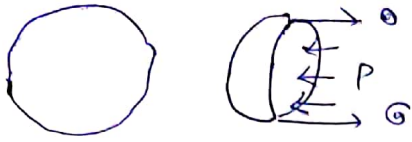
(3) capillary rise or falling

Pond

It is denoted by ( $\sigma$ )

## Pressure calculations

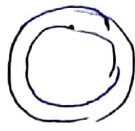
(1) water droplet  $\rightarrow$  (2) soap hollow bubble  $\rightarrow$  (3) Liquid jet  $\rightarrow$



$$P_F = P \times \frac{\pi}{4} \times d^2$$

$$S_F = \sigma \times \pi d$$

$$\Rightarrow P = \frac{4\sigma}{d}$$



It is having two surfaces

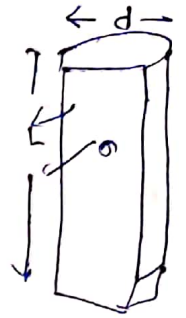
$$P = \frac{8\sigma}{d}$$

considering a cylindrical liquid jet

$$P_F = P \times d \times L$$

$$S_F = 2L \times \sigma$$

$$\Rightarrow P = \frac{2\sigma}{d}$$



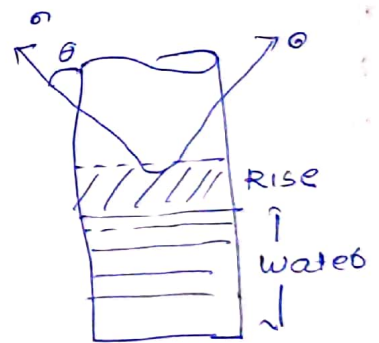
The unit is taken as N.m and dimension is  $[M^1 T^{-2} L^0]$

## capilarity

capilarity is a phenomenon by which a liquid rises / fall inside a narrow tube above or below the general level. This phenomenon is due to the combined effect of cohesion and adhesion of liquid particle.

$$h = \frac{4 \times \sigma \times \cos \theta}{\rho \times g \times d}$$

For water  $\theta = 0^\circ$   $h = \frac{4\sigma}{\rho \times g \times d}$



In case of mercury there is capilarity fall or depression. For this angle of contact is  $140^\circ$

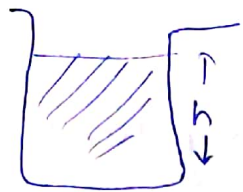
When the cohesive force  $>$  adhesive  $\rightarrow$  Rise  
 " " "  $<$  adhesive  $\rightarrow$  Fall

## Pressure

When a fluid is contained inside a vessel it exerts force at all the points on the side and bottom. This force per unit area is called pressure.

$$P = \frac{\text{Force}}{\text{Area}} = N/m^2$$

## Pressure head of liquid



$$P = \rho \times g \times h$$

$$\Rightarrow h = \frac{P}{\rho g} \text{ is called pressure head}$$

$h = \text{depth}$

$\rho = \text{density}$

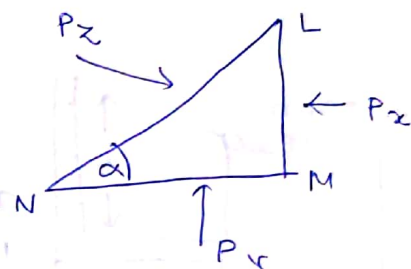
From this we observe that intensity of pressure is directly proportional to depth.

## Pascal's Law

The intensity of pressure at any point in a liquid at rest is same in all the direction.

$$P_x \times LM = P_y \times NM = P_z \times LN$$

$$P_x = P_y = P_z$$



## Different types of pressure

- (1) **atmospheric pressure**  $\rightarrow$  The atmospheric air exerts a normal pressure upon all the surface in contact is called as atmospheric pressure. It is also known as Barometric pressure. at sea level is called standard atm pressure.
- (2) **gauge pressure**  $\rightarrow$  It is the pressure measured with the help of pressure measuring device above the atmospheric pressure.
- (3) **vacuum pressure**  $\rightarrow$  When the pressure measurement takes place below the atmospheric pressure is called vacuum pressure.

The S-I unit of pressure is  $N/m^2$  also called pascal.

$$1 \text{ bar} = 10^5 \text{ pascal}$$

$$1 \text{ atm} = 1.01325 \text{ kPa}$$

$$= 760 \text{ mm of Hg}$$

$$= 10.33 \text{ m of } H_2O$$

$$= 101.325 \text{ kPa}$$

(4) absolute pressure =  $P_{atm} + P_{gauge}$   
 $= P_{atm} - P_{vacuum}$

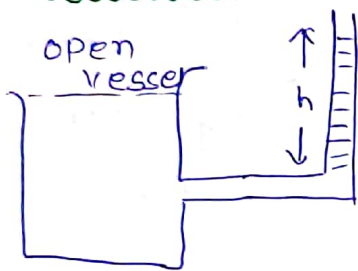
Measurement of pressure

Pressure can be measured by the following devices

(1) Manometers

manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column. It is again divided into various types →

(1) Piezometer



$P = \rho \times g \times h$

It measures gauge pressure only and when it is open to the atmosphere it also experiences atmospheric pressure

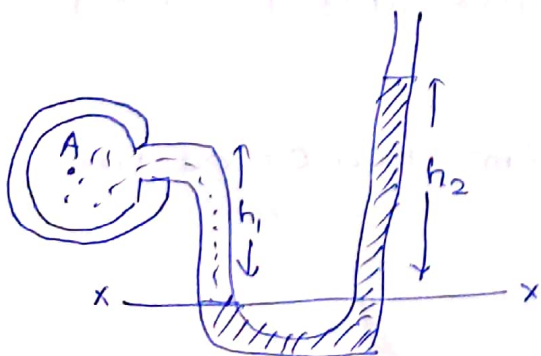
$P_{total} = P_{atm} + \rho \times g \times h$

(2) U-tube manometer

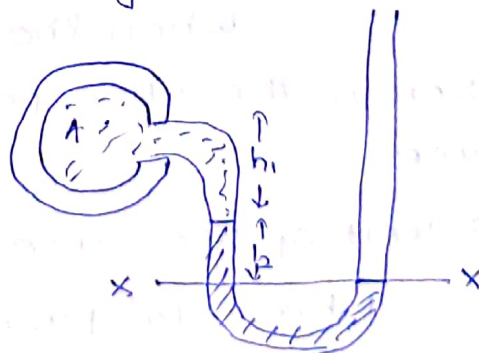
Piezometer can't be employed for large pressure so we use it which is consisting of a glass tube bent into U-shape. one end is connected to a point where pressure is being measured. other end remains open to the atmosphere.

(1) Positive pressure →

(2) negative pressure →



$P_A + \rho_1 g h_1 = \rho_2 \times g \times h_2$



$P_A + \rho_1 g h_1 + \rho_2 g h_2 = 0$

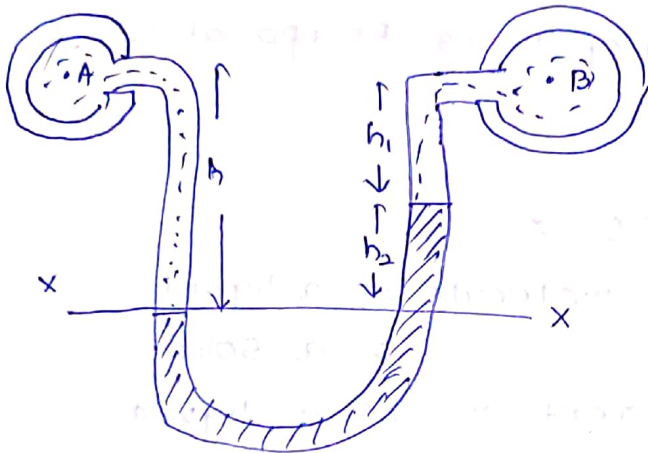
Pressure in left limb = Pressure in Right limb

# Differential Manometers

It is used to measure the difference in pressure between the two points in a pipeline or in two different pipes. It is again two types →

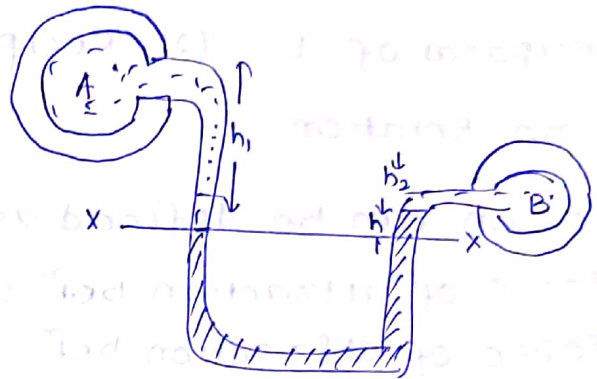
## (1) U-tube differential Manometers →

case-1 (Same level)



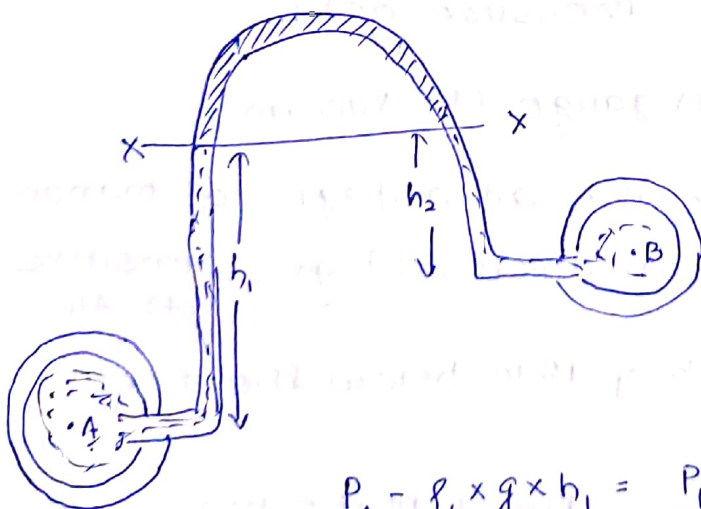
$$P_A + \rho_1 \times g \times h = P_B + \rho_1 \times g \times h_1 + \rho_2 \times g \times h_2$$

case-2 (Different level)



$$P_A + \rho_1 \times g \times h_1 + \rho_1 \times g \times h = P_B + \rho_2 \times g \times h + \rho_1 \times g \times h_2$$

## (2) Inverted U-tube manometers →



$$P_A - \rho_1 \times g \times h_1 = P_B - \rho_2 \times g \times h_2$$

~~8466~~  
~~7928~~  
~~31824~~

~~279~~  
~~290~~  
~~(23)~~

## Advantages of using manometers

- (1) very little maintenance required
- (2) highly sensitive and gives accurate result
- (3) But measures very low pressure only

## Problems of chapters (1) and (2)

- (a) Elasticity of fluids can be measured by  $\rightarrow$   
(1) E (2)  $G_1$  (3) K (4) NONE
- (b) Bulk modulus of elasticity \_\_\_\_\_ with increase in pressure.  
(1) increases (2) decreases (3) no change (4) Remains constant
- (3) compressibility can be defined as the  $\rightarrow$   
(1) Reciprocal of E (2) Reciprocal of K (3) Reciprocal of  $G_1$   
(4) no Relation
- (4) cohesion can be defined as the  $\rightarrow$   
(1) Force of attraction bet<sup>n</sup> same molecule of a liquid.  
(2) Force of attraction bet<sup>n</sup> " " of a solid  
(3) " " " different " of a liquid.  
(4) NONE of above
- (5) Any pressure measured above the absolute zero temp  
is called  
(1) gauge pressure (2) atmospheric P (3) vacuum P (4) Both (1) & (2)
- (6) Piezometers measures \_\_\_\_\_ pressure only.  
(1) absolute (2) atmospheric (3) gauge (4) vacuum
- (7) Which of the following is/are the advantages of manometers  
(1) good accuracy (2) little maintenance (3) highly sensitive  
(4) All
- (8) Find the pressure at a depth of 15 m below the free  
surface of water?
- (9) A plate of 5 cm distance from a fixed plate moves at  
1.2 m/second and requires a force of  $2.2 \text{ N/m}^2$  to maintain  
it's speed. Find the viscosity of the fluid bet<sup>n</sup> plates?
- (10) A soap bubble of 62.5 mm in diameter has an internal  
pressure of  $20 \text{ N/m}^2$  Find the surface tension of  
the bubble?

# Hydrostatic Force on Surfaces

Hydrostatic means the study of pressure exerted by a fluid at rest. The direction of pressure is always perpendicular to the surface.

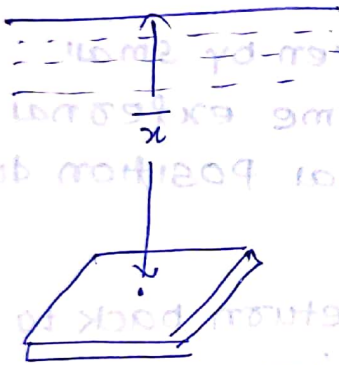
## Total pressure and centre of pressure

Total pressure is defined as the force exerted by static fluid on a surface when the fluid comes in contact with the surface.

Centre of pressure is defined as the point of application of the total pressure on the surface.

### Case-1 (Horizontal)

As we know  $P = \frac{F}{A}$



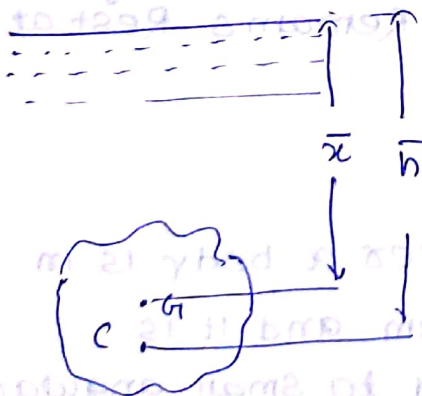
$$\Rightarrow F_p = P \times A$$

$$= \rho \times g \times h \times A$$

$$= \rho \times g \times \bar{x} \times A$$

centre of pressure =  $\bar{x}$

### Case-2 (Vertical)



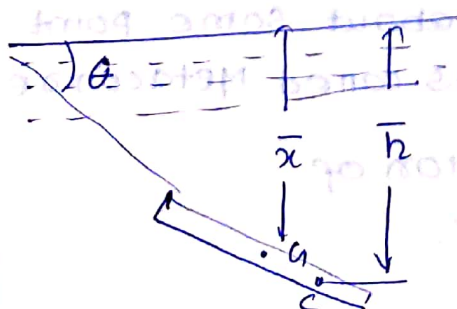
same equation

$$F_p = \rho \times g \times \bar{x} \times A$$

centre of pressure =  $\bar{h}$

$$\bar{h} = \frac{I_G}{A \cdot \bar{x}} + \bar{x}$$

### Case-3 (Inclined)



same equation

$$F_p = \rho \times g \times h \times A$$

$$= \rho \times g \times \bar{x} \times A$$

centre of pressure

$$\bar{h} = \frac{I_G \sin^2 \theta}{A \cdot \bar{x}} + \bar{x}$$

## Buoyancy & Floatation

Whenever a body is immersed fully or partially in a fluid it is subjected to an upward force which tends to lift up the body is called Buoyancy Force.

The point of application of the force of Buoyancy on the body is called centre of buoyancy.

## Archimedes Principle

When a body is immersed in a fluid either wholly or partially, it is lifted up by a force which is equal to the weight of the fluid displaced by the body.

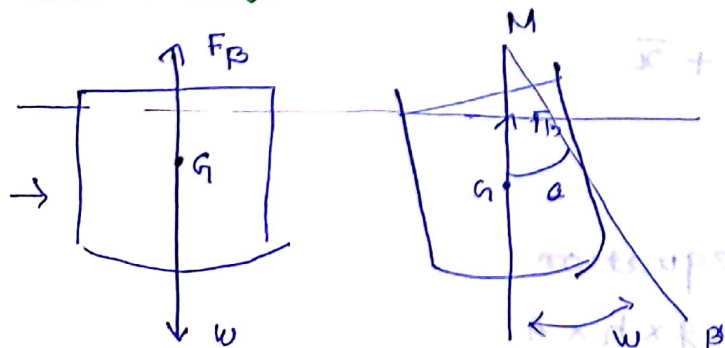
## Types of Equilibrium of floating bodies

**Stable equilibrium** → Whenever a body is given by small angular displacement or tilted slightly by some external force then it returns back to the original position due to some external force.

**Unstable equilibrium** → If the body does not return back to its original position is called unstable equilibrium.

**Neutral equilibrium** → If a body when given by small angular displacement occupies a new position and remains rest at this new position.

## Metacentre



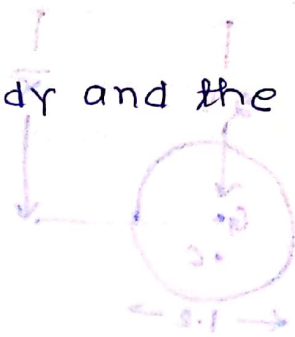
Whenever a body is in equilibrium and it is subjected to small angular displacement it starts oscillating about some point here (M) is called Metacentre.

It is also be defined as the intersection of C.G of the body and centre of buoyancy.

## Metacentric height

The distance between C.G of a body and the Metacentre is called metacentric height.

- 1) stable equilibrium  $\rightarrow$  M above G
- 2) unstable "  $\rightarrow$  M below G
- 3) Neutral  $\rightarrow$  M and G coincides



(\*) It can be determined by both Analytical and experimental methods. The Metacentre Remains same for all displacement

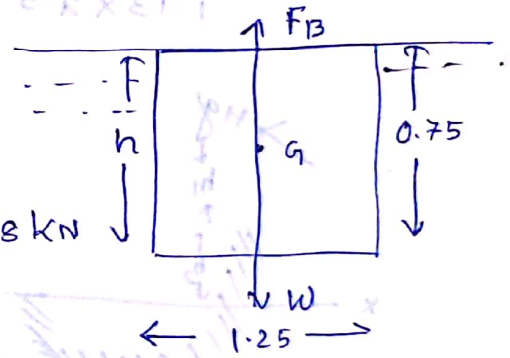
## Question of chapters (3) and (4)

(1) A wooden block of width 1.25 m, depth 0.75, Length 3.0 m is floating in water. Specific gravity of wood is 0.4  $\text{KN/m}^3$ . Find (1) volume of water displaced (2) position of C.O.B

Ans  $\rightarrow$  Volume =  $L \times b \times h = 1.25 \times 0.75 \times 3 = 2.8125 \text{ m}^3$

Weight =  $\rho \times g \times V$

$= S.W \times V = 0.4 \times 2.8125 = 1.125 \text{ kN}$



Volume of water displaced  $\rightarrow$

$W_{\text{Block}} = \text{weight of water displaced}$

$1.125 = \frac{\text{weight of water displaced}}{\text{weight density}} = \frac{1.125}{9.81} = 0.1147$

Centre of buoyancy  $\rightarrow$

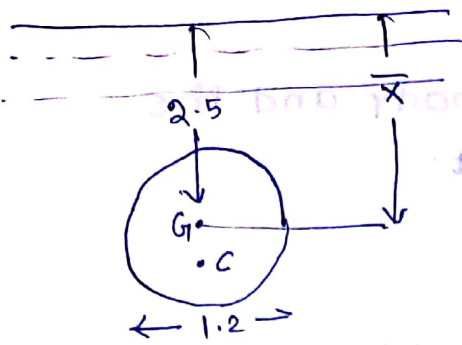
Volume of wooden block in water = volume of water displaced

$1.25 \times 3 \times h = 1.147$

$\Rightarrow h = \frac{1.147}{1.25 \times 3} = 0.302$

Centre of buoyancy =  $\frac{h}{2} = \frac{0.302}{2} = 0.151$

(2)



$d = 1.5 \text{ m}$  placed vertically in water such a way that centre of plate is  $2.5 \text{ m}$  below the free surface

so we get  $\bar{x} = 2.5$   
 $A = \pi R^2 = \pi \times \left(\frac{1.2}{2}\right)^2 = 1.13$

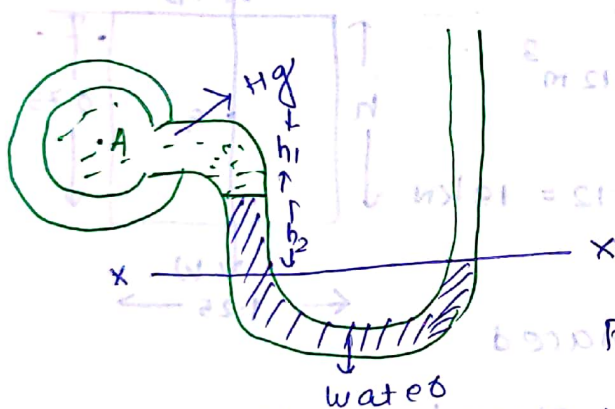
Total pressure =  $\rho \times g \times \bar{x} \times A = 1000 \times 9.81 \times 2.5 \times 1.13$   
 $= 27.7 \text{ kN}$

Centre of pressure  $\bar{h} = \frac{I_G}{A \cdot \bar{x}} + \bar{x}$

$I_G = \frac{\pi d^4}{64} = \frac{\pi}{64} \times (1.2)^4 = 0.1018$

$\bar{h} = \frac{0.1018}{1.13 \times 2.5} + 2.5 = 2.536 \text{ m}$

(3)



U-tube manometer containing

$h_1 = 2 \text{ m}$      $h_2 = 4 \text{ m}$

$P_A + \rho_1 \times g \times h_1 + \rho_2 \times g \times h_2 = 0$

$P_A + 13.6 \times 10^3 \times 9.81 \times 2 + 1000 \times 9.81 \times 4$   
 $\Rightarrow P_A = -(13.6 \times 10^3 \times 9.81 \times 2 + 1000 \times 9.81 \times 4)$

(\*) Note:

Density of water =  $1000 \text{ kg/m}^3$

Density of Mercury =  $13.6 \times 10^3 \text{ kg/m}^3$

If specific gravity is given

then density =  $S \cdot g \times 1000$

Bulk modulus of elasticity

It is denoted by  $k = \frac{dp}{-\left(\frac{dv}{v}\right)}$  decrease in volume so  $-(dv)$

compressibility is the property of fluid by virtue of which there is change in volume due to the application of external force. It is also defined as Reciprocal of  $k$ .

# Fluid Kinematics

here we have to Analyze the velocity and acceleration of the fluid flow. here the Analysis is divided into two methods

(1) **Lagrangian Method** → here the observers concentrate on the movement of the single particle. The path taken by the particle and change in velocity and acceleration is calculated. In the cartesian co-ordinate of 3-D space we have

$$x = f_1(a, b, c, t) \quad y = f_2(a, b, c, t) \quad z = f_3(a, b, c, t)$$

$$\text{Velocity component } u = \frac{\partial x}{\partial t}, \quad v = \frac{\partial y}{\partial t}, \quad w = \frac{\partial z}{\partial t}$$

$$\text{Acceleration component } a_x = \frac{\partial^2 x}{\partial t^2}, \quad a_y = \frac{\partial^2 y}{\partial t^2}, \quad a_z = \frac{\partial^2 z}{\partial t^2}$$

$$\text{Resultant velocity } v = \sqrt{u^2 + v^2 + w^2}$$

$$\text{Resultant acceleration} = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

(2) **Eulerian Method** → here the observers concentrate at a particular point in a fluid system and at that particular point velocity and acceleration can be studied.

$$u = f_1(x, y, z, t) \quad v = f_2(x, y, z, t) \quad w = f_3(x, y, z, t)$$

$$\text{As we know } u = \frac{dx}{dt}, \quad v = \frac{dy}{dt}, \quad w = \frac{dz}{dt}$$

$$a_x = \frac{du}{dt} = u \times \frac{\partial u}{\partial x} + v \times \frac{\partial u}{\partial y} + w \times \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = \frac{dv}{dt} = u \times \frac{\partial v}{\partial x} + v \times \frac{\partial v}{\partial y} + w \times \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = \frac{dw}{dt} = u \times \frac{\partial w}{\partial x} + v \times \frac{\partial w}{\partial y} + w \times \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

$$\text{Again Resultant velocity } v = \sqrt{u^2 + v^2 + w^2}$$

$$\text{Resultant acceleration } a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

## Types of Fluid Flow

(1) **Steady vs unsteady** → when the flow parameters such as velocity, acceleration, pressure does not change with time is called steady flow otherwise it is called as un-steady flow.

(2) uniform vs non-uniform  $\rightarrow$  The type of flow in which velocity at any given time does not change with respect to the space is called uniform flow. If the velocity changes is called non-uniform flow.

(3) one, two or three dimensional flow  $\rightarrow$  When the flow parameters are the function of single co-ordinate is called one-D flow. Similarly when the flow is in other dimension is called 2D and 3D-flow.

(4) compressible vs incompressible flow  $\rightarrow$  The type of flow in which the density of the fluid changes from point to point is called compressible flow. If the density remains constant is called incompressible flow.

(5) Laminar vs Turbulent flow  $\rightarrow$  Laminar flow is the type of flow in which the fluid particles do not cross one another and moved in a well defined path.

A Turbulent flow is the type of flow in which fluid particle moves in a zig-zag pattern.

To evaluate between laminar and turbulent flow Reynolds number is the parameter.

$Re < 2000$  (Laminar)  $Re > 4000$  (Turbulent)

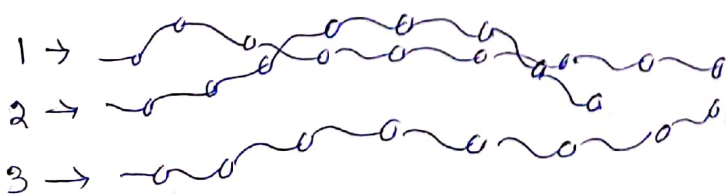
(6) Rotational vs irrotational flow  $\rightarrow$  A flow is said to be rotational if the fluid particle while moving in their direction of flow rotates about their mass centre.

ex  $\rightarrow$  Motion of liquid in a rotating tank.

If they does not rotate we called as irrotational flow.

### Types of Flow Lines

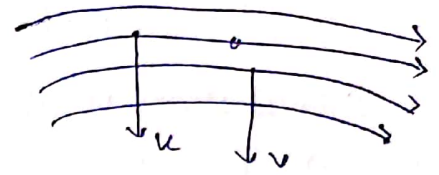
(1) path line  $\rightarrow$  It is the path followed by the fluid particle in motion. This is the curve for a 3-D space.



(2) **Stream line** → It is defined as an imaginary line drawn across the Flow Field so that the tangent at any point gives the direction of velocity at that point

Equation of stream line is given as

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$



- 1) Two stream line never intersect each other, nor cross each other.
- 2) There is no fluid mass accumulation in a stream line.
- 3) For steady flow it remains invariant but for unsteady it remains changed.

(3) **Streak line** → It is a curve which gives an instantaneous picture of the location of the fluid particles which have passed through a given point.

ex → (1) smoke coming out from chimney

### Rate of flow / Discharge

It is the quantity of liquid flowing per second through a section of pipe or channel.

$$Q = A \times V = \text{m}^3/\text{second}$$

$\downarrow$        $\downarrow$   
 Area    velocity

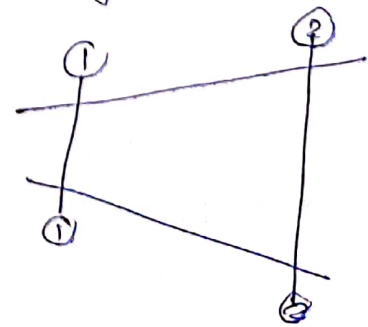
### Continuity Equation

It is based on the principle of conservation of mass. It states that if no fluid is added or removed from the pipeline in any length, then the mass passing across the different section shall be same.

$$\rho_1 \times A_1 \times V_1 = \rho_2 \times A_2 \times V_2 \quad (\text{compressible})$$

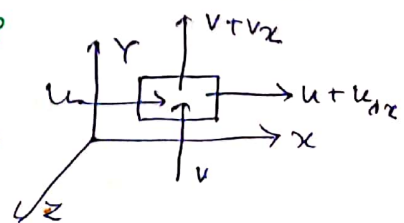
If the fluid is incompressible

$$A_1 \times V_1 = A_2 \times V_2$$



### Continuity Equation in Cartesian co-ordinate

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \text{When the flow is incompressible and steady}$$

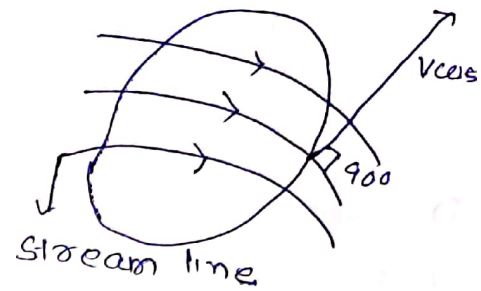


## Circulation and Vorticity

Inside a closed curve in a 2-D Flow Field Let the curve cut the stream line at point P and tangent gives the direction of velocity.

Circulation is defined as the line integral of the tangential velocity about a closed path.

$$\Gamma = \oint v_{\text{tangential}} \cdot ds$$



When the circulation is considered per unit Area is called vorticity ( $\Omega$ ).  $\Omega = \frac{\Gamma}{A}$

If the flow possesses vorticity then the fluid elements are rotational.

## Velocity Potential Function

The velocity potential function is defined as scalar function of space and time such that its negative derivative w.r.t. to any direction gives the fluid velocity in that direction.

It is denoted by  $\phi$  (Phi) Let  $\phi = f(x, y, z, t)$

$$u = -\frac{\partial \phi}{\partial x}, \quad v = -\frac{\partial \phi}{\partial y}, \quad w = -\frac{\partial \phi}{\partial z}$$

Now putting these values in the continuity Equation we get  $\rightarrow$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\Rightarrow \frac{\partial}{\partial x} \left( -\frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( -\frac{\partial \phi}{\partial y} \right) + \frac{\partial}{\partial z} \left( -\frac{\partial \phi}{\partial z} \right) = 0$$

$$-\left( \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \right) = 0 \quad \text{is called Laplace equation}$$

If the velocity potential function satisfies Laplace equation it represents the possible case of steady and incompressible, rotational flow.

**Equipotential line**  $\rightarrow$  It is the line along which the  $\phi = \text{constant}$ .

$$\text{where } \frac{dy}{dx} = \text{slope of equipotential line } d(\phi) = 0$$

## Stream Function

It is the function of space and time such that its partial derivative w.r. to any direction gives the velocity component at right angle to this direction. denoted by ( $\psi$ )

$$\psi = f(x, y, t)$$

For 2-D flow continuity equation is

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial}{\partial x} \left( \frac{\partial \psi}{\partial y} \right) + \frac{\partial}{\partial y} \left( -\frac{\partial \psi}{\partial x} \right)$$

$$\Rightarrow \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial x \partial y} = 0$$

hence it is a possible case of fluid flow.

(\*) on any streamline  $\psi$  is constant everywhere.

(\*) If the flow is continuous the flow around any path is zero.

## Relation between stream function and velocity potential function

For velocity potential function ( $\phi$ ) = constant

$$\text{slope} = \frac{dy}{dx} = \frac{\frac{\partial \phi}{\partial x}}{\frac{\partial \phi}{\partial y}} = -\frac{u}{v} = \frac{v}{u}$$

For stream function ( $\psi$ ) = constant

$$\text{slope} = \frac{dy}{dx} = \frac{\frac{\partial \psi}{\partial x}}{\frac{\partial \psi}{\partial y}} = -\frac{v}{u} = -\frac{v}{u}$$

$$\text{Product of slope} = \frac{v}{u} \times -\frac{v}{u} = -1$$

This shows that stream line and equipotential line intersect each other orthogonally.

## Flow Nets

A grid obtained by drawing a series of stream line and equipotential line is known as Flow net.

## Special Notes

1) Rotational Flow  $\rightarrow$  If the parameters are given in vector form ( $\text{curl } \vec{v} = 0$ ) is said to be irrotational flow.

$$\text{curl } \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} \quad \text{where } \vec{v} = u\hat{i} + v\hat{j} + w\hat{k}$$

The Rotational component along different directions are given by

$$w_x = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \quad w_y = \frac{1}{2} \left( \frac{\partial w}{\partial z} - \frac{\partial u}{\partial x} \right) \quad w_z = \frac{1}{2} \left( \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)$$

(2) TO CHECK FLOW  $\rightarrow$

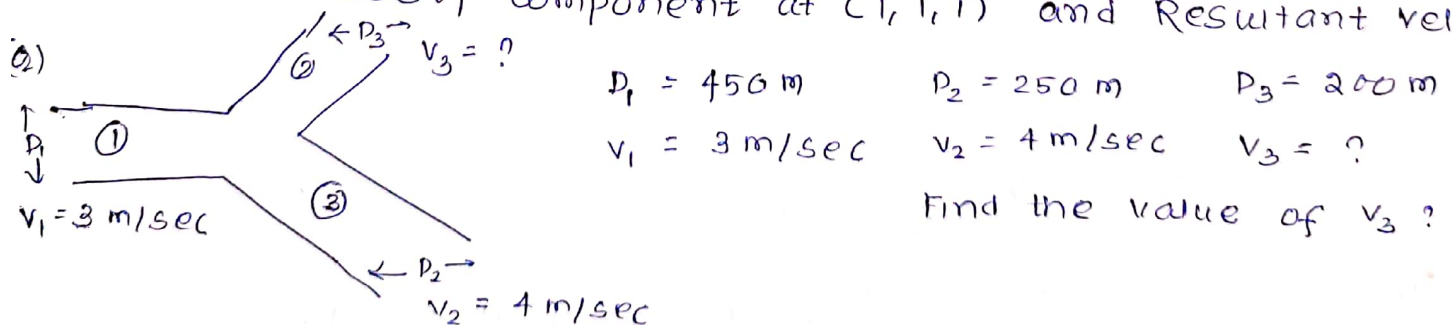
When a flow satisfies continuity equation we can say that the flow is incompressible.

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## Questions

- (Q) If  $u = xy$  and  $v = 2yz$  check whether it is 2-D flow or not.
- (Q) If  $u = 2x$ ,  $v = 4y$ ,  $w = 6z$  write the equation of stream line
- (Q) For a vector function  $v = 2x\hat{i} + 3y\hat{j} + 4z\hat{k}$  check whether it is rotational or not?

(Q) In a fluid flow  $v = (3x + 2y)\hat{i} + (2z + 3x^2)\hat{j} + (2t - 3z)\hat{k}$   
Find the velocity component at  $(1, 1, 1)$  and resultant velocity



(Q) In a fluid flow  $u = x^3 - y^3$  and  $v = 3y^2 - 4x$  and  $w = 4z - 8$   
check whether it is incompressible or not?

(Q) If  $u = y^2$  and  $v = -3x$   
Find the value of  $w_x = ?$

# Fluid Dynamics

In Dynamics we Analyze the various forces which held responsible for it's motion. The fluid is assumed to be incompressible and non-viscous. The Basic equation which we are going to Analyze are (i) continuity equation (ii) Energy equation (iii) Impulse-momentum equation

## Different types of head/ Energy of a liquid in motion

(1) Potential head/ Potential Energy  $\rightarrow$  This is due to the position or configuration above some suitable datum line. (2)

(2) velocity head/ kinetic Energy  $\rightarrow$  This is due to the velocity of flowing liquid as  $\frac{v^2}{2g}$ .

(3) Pressure Energy/ Pressure head  $\rightarrow$  This is due to the pressure of liquid denoted as  $h = \frac{P}{\rho \times g}$

Total head/ Energy = (H) =  $\frac{P}{\rho g} + \frac{v^2}{2g} + z$

It can also be taken as Energy (E) = P.E + K.E + P<sub>s</sub>.E

## Bernoulli's Equation

In an ideal incompressible fluid when the flow is steady and continuous the sum of pressure head, velocity head and datum head is constant along a streamline.

$$\frac{P}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

For two points in a fluid flow we can write the equation as

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

Various assumptions are  $\rightarrow$

- 1) Flow is along streamline and one-dimensional.
- 2) velocity must be uniform
- 3) only force acting are pressure and gravity

(\*) Every term in Bernoulli's equation have the unit as meter.

(\*) It deals with the conservation of Energy.

## Euler's Equation of Motion

During the Analysis of Euler's Equation the various forces we are considering for our analysis is

- (1) pressure force
- (2) gravity or body force
- (3) inertia force

we are getting same Bernoulli's Equation

$$\frac{dp}{\rho} + v \cdot dv + g \cdot dz = 0 \dots \dots \text{(differential Equation)}$$

Now integrating the above equation

$$\frac{1}{\rho} \int dp + \int v dv + g \int dz$$
$$= \frac{P}{\rho} + \frac{v^2}{2} + gz$$

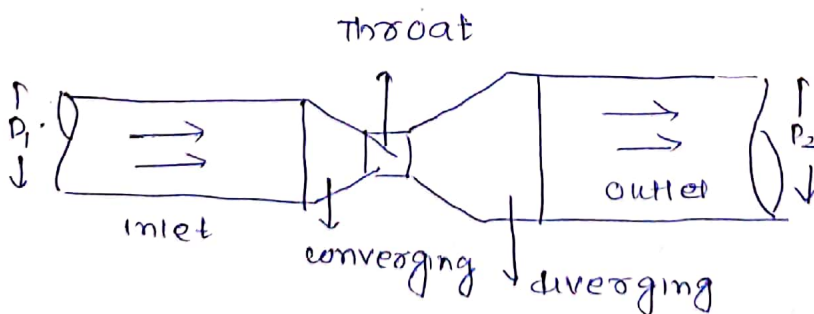
Now divide  $g$  on all terms of the equation we get

$$\frac{P}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

## Application of Bernoulli's Equation

### (1) Venturimeter

It is used to measure the Rate of discharge along a pipeline and also fixed permanently to measure flow rate. Different types of venturimeter are there like horizontal, vertical and inclined.



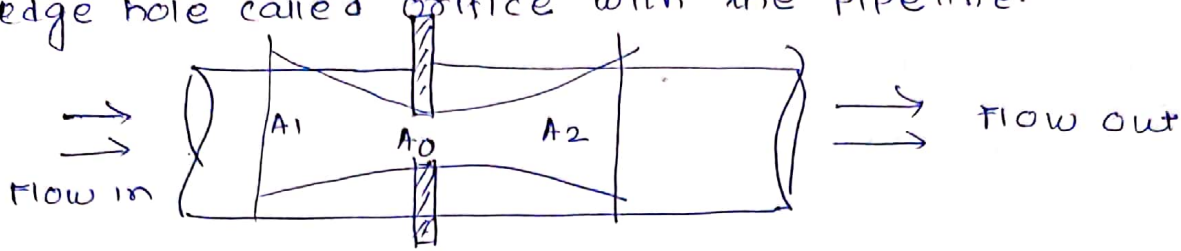
Length of diverging section always greater than converging section.

$$Q_{\text{actual}} = C_d \times \frac{A_1 \times A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$$

$C_d$  = coefficient of discharge of venturimeter

## (2) Orifice meter

It is also used to measure the discharge of fluid through a pipe. It works on the same principle as venturimeter. It is having a flat circular plate having a edge hole called orifice with the pipeline.

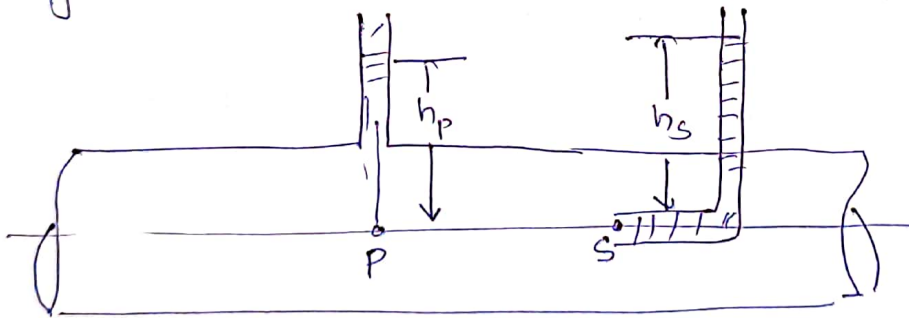


$$Q_{\text{actual}} = C_d \times \frac{A_0 \times A_1}{\sqrt{A_1^2 - A_0^2}} \times \sqrt{2gh}$$

## (3) Pitot tube

It is one of the most accurate device for measuring velocity. It works on the principle that if the velocity of flow at a point is zero the pressure increased to conversion of K.E to pressure Energy.

It is consisting of a glass tube in the form of 90° bend of short length pipe. It is placed in the flow such that a stagnation point is created immediately after the opening.



$$v = \sqrt{2gh}$$
$$= \sqrt{2 \times g \times (h_s - h_p)}$$

It also worked on the principle of Bernoulli's theorem.

## Hydraulic co-efficients

(1) Co-efficient of contraction  $\rightarrow$

$$C_c = \frac{\text{Area at vena-contract}}{\text{Area of the orifice}} = \frac{a_c}{a}$$

(2) Co-efficient of velocity ( $C_v$ )  $\rightarrow$

$$C_v = \frac{\text{Actual velocity}}{\text{Theoretical velocity}} = \frac{V}{\sqrt{2gh}}$$

(3) Co-efficient of discharge ( $C_d$ )  $\rightarrow$

$$C_d = \frac{\text{Actual discharge}}{\text{Theoretical discharge}} = \frac{Q_{\text{actual}}}{Q_{\text{theoretical}}}$$

$$C_d = C_c \times C_v$$

~~\*\*\*~~

### Questions

(1) Piezometric head is the summation of

- (a) velocity & pressure head (b) pressure and elevation head  
(c) velocity & elevation head

(2) The water is flowing through a taper pipe having diameters 300 mm and 150 mm at section (1) and (2). The discharge through the pipe is 40 litres/sec. The section (1) is 10 m above datum and section 2 is 6 m above datum. Find the pressure at 2 if pressure at section 1 is 400 kN/m<sup>2</sup>?

(3) If the  $C_v$  of a orificemeter is 0.3 and  $C_d$  is 0.6 Find the value of  $C_c$ ?

(4) Pitot tube measures

- (1) pressure (2) Energy (3) Velocity (4) Force

(5) The pressure at the stagnation point is

- (1) zero (2) Maximum (3) Minimum (4) Not applicable

(6) To measure high rate of discharge along a pipeline

- (1) orificemeter (2) Pitot tube (3) venturimeter (4) All

(7) The  $C_d$  value of orificemeter is \_\_\_\_\_ than venturimeter

- (1) greater (2) less (3) equal (4) none

# Concept of Laminar and Turbulent Flow

$$\text{Reynolds number} = \frac{\text{Inertia Force}}{\text{viscos force}}$$

As we know  $Re < 2000$  (Laminar)  
 $Re > 4000$  (Turbulent)

Between 2000 to 4000 indicate the transition from L to T

$$Re = \frac{\rho \times v \times D}{\mu} = \frac{v \times D}{\frac{\mu}{\rho}} = \frac{v \times D}{\nu}$$

Due to Friction in the pipe line there is always head loss which is calculated as  $h_f = \frac{f \times L \times v^2}{2 \times g \times d}$

$$f = \text{friction co-efficient} = \frac{64}{Re} \text{ (Laminar)}$$

In case of turbulent flow  $f =$

In case of turbulent flow due to the zig-zag motion the particles are completely mix with one another.

$$\text{The Laminar flow } \frac{v_{\max}}{v_{\text{avg}}} = 1.5$$

$$\text{In case of pipe flow } \frac{v_{\max}}{v_{\text{Avg}}} = 2$$

## Hydraulic Turbine

Turbines are the hydraulic machines which converts the hydraulic energy into Mechanical energy. Thus this mechanical energy is used to generate electrical energy.

### Classification

(1) Impulse vs Reaction Turbine  $\rightarrow$  If at the inlet of the turbine the entire available energy gets converted to kinetic energy it is called Impulse Turbine. here it works on the principle of impulse.

If at the inlet of the turbine the water passes both kinetic energy and pressure energy is called Reaction turbine.

(2) according to the direction of flow through Runners  $\rightarrow$



If the water flows along the tangent of the runner the turbine is called tangential flow turbine. If it is in the radial direction is called Radial Flow turbine.

If the water flows outward to inward radially then it is called inward Radial Flow. If it flows from inward to outward the turbine is called outward Radial Flow turbine.

### 3) Axial Flow vs Mixed Flow Turbine →

If the water flows through the runner along the direction parallel to the axis of rotation of the runner the turbine is called axial flow turbine.

If the water flows in radial direction but leaves the runner direction parallel to the axis of rotation is called mixed flow turbine.

Further it is again divided into high, low and medium head turbine and w.r.to specific speed it is divided into high, medium and low specific speed turbine.

### Efficiencies of Turbine

(1) hydraulic efficiency →  $\eta_H = \frac{\text{Runner Power}}{\text{water power}}$

(2) Mechanical efficiency →  $\eta_m = \frac{\text{Shaft Power}}{\text{Runner Power}}$

(3) overall efficiency →  $\eta_o = \eta_H \times \eta_m$   
 $= \frac{\text{Shaft Power}}{\text{water power}}$

Water power =  $\frac{\rho \times g \times Q \times H}{1000}$  kW

Shaft power (P) =  $\frac{2\pi NT}{60,000}$  kW

### Working of a Turbine

Pelton wheel turbine is a purely impulse turbine and it is tangential flow turbine. The water strikes the bucket along the tangent of the runner. The energy available at inlet is kinetic energy. The pressure at inlet and outlet is atmospheric pressure. It is used for high head and discharge.

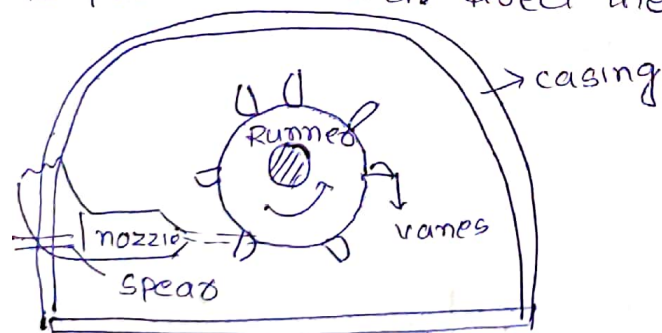
The water from the Reservoir flows through the penstock at the outlet of which nozzle is fixed. The nozzle increases the K.E of water flowing through the penstock. At the outlet of the nozzle the water comes out in the form of jets and strikes the buckets of the runner. The main parts are  $\rightarrow$

(1) **Nozzle and Flow Regulating arrangement**  $\rightarrow$  The controlling of water striking the vanes is controlled by spears.

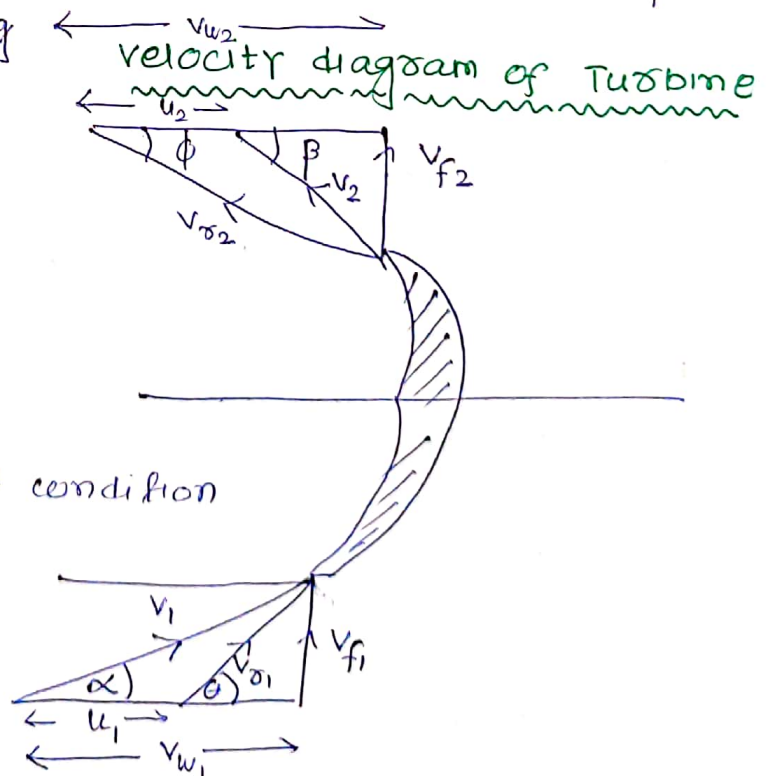
(2) **Runner with buckets**  $\rightarrow$  It consists of a circular disc on the periphery of which no. of buckets space are fixed. The shape of the bucket is double hemispherical cup or bowl. Each bucket is divided into two symmetrical parts. The blades are such that the jet gets deflected through  $160^\circ$  to  $170^\circ$ . Buckets are made from cast iron and stainless steel.

(3) **Casing**  $\rightarrow$  The function of the casing is to prevent the splashing of water and also safeguard against accidents. It does not perform any hydraulic function.

(4) **Breaking Jet**  $\rightarrow$  When the nozzle is completely closed still the runner goes on revolving due to inertia force. To stop it a small nozzle is provided which direct the jet of water on the back of vanes.



- $v_1$  = absolute velocity
- $v_{r1}$  = Relative velocity
- $u_1$  = blade speed
- $v_{f1}$  = Flow velocity
- $v_{w1}$  = whirl velocity
- $\alpha$  = nozzle angle
- $\theta$  = blade angle



Similarly for outlet all having equal names

It's Analysis is Based upon the impulse-momentum principle

$$\text{Mass Flow Rate at inlet} = \rho \times a \times v_1$$

$$\text{momentum at inlet} = \rho \times a \times v_1 \times v_{w1}$$

$$\text{Moment of momentum} = \rho \times a \times v_1 \times v_{w1} \times R_1 \quad (\text{inlet})$$

$$\text{Similarly moment of momentum at outlet} = - \rho \times a \times v_2 \times v_{w2} \times R_2$$

$$\text{Rate of change of Momentum} = \rho \times a \times v_1 \times v_{w1} \times R_1 - \rho \times a \times v_2 \times v_{w2} \times R_2$$

$$\text{From the Blade speed } (u_1) = v_1 \times R_1 \text{ and } u_2 = v_2 \times R_2$$

$$\text{So Rate of change of Momentum} = \rho \times a \times u_1 \times v_{w1} - \rho \times a \times u_2 \times v_{w2}$$

$$\text{Torque} = \text{Rate of change of momentum}$$

$$= \rho \times a (v_{w1} u_1 + v_{w2} u_2)$$

$$\text{Work done/sec} = \text{Torque} \times \omega$$

$$= \rho \times a (v_{w1} u_1 + v_{w2} u_2) \times \omega$$

$$= \rho \times a \times v_{w1} \times v_1 \times R_1 \times \omega + \rho \times a \times v_{w2} \times R_2 \times v_2 \times \omega$$

$$= \rho \times a \times v_{w1} \times v_1 \times u_1 + \rho \times a \times v_{w2} \times v_2 \times u_2$$

$$= \rho \times a \times v_1 (v_{w1} u_1 + v_{w2} u_2)$$

$$\text{Runner power} = \frac{\rho \times a \times v_1 (v_{w1} u_1 + v_{w2} u_2)}{1000} \text{ kW}$$

### 1) Pelton wheel Turbine

$$\text{here } \alpha = 0^\circ$$

$$R_1 = R_2$$

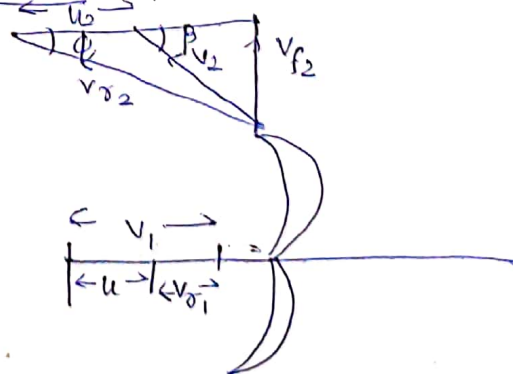
$$u_1 = u_2 = u$$

$$v_1 = v_{w1}$$

$$\text{R.P} = \frac{\rho \times a \times v_1 (v_{w1} u_1 + v_{w2} u_2)}{1000} \text{ kW}$$

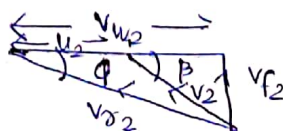
From the outlet velocity triangle  $\rightarrow$

$$v_{w2} = v_{r2} \cos \phi - u_2$$



From the diagram

$$v_{r1} = v_1 - u$$



$$V_{w2} = (V_1 - u) \times \cos \phi - u$$

Now putting it in the equation of Runner's power

$$\begin{aligned} R.P &= \frac{\rho \times a \times V_1 (V_{w1} + V_{w2}) \times u}{1000} \text{ Kw} \\ &= \frac{\rho \times a \times V_1 (V_1 + (V_1 - u) \times \cos \phi - u) \times u}{1000} \\ &= \frac{\rho \times a \times V_1 (V_1 + V_1 \cos \phi - u \cos \phi - u) \times u}{1000} \\ &= \frac{\rho \times a \times V_1 ((1 + \cos \phi) V_1 - (1 + \cos \phi) u)}{1000} \\ &= \frac{\rho \times a \times V_1 \times (1 + \cos \phi) (V_1 - u) \times u}{1000} \text{ Kw} \end{aligned}$$

If the blade is smooth  $\frac{V_{\theta 2}}{V_{\theta 1}} = 1$

$$V_{\theta 2} = V_{\theta 1}$$

$$\begin{aligned} V_{w2} &= V_{\theta 2} \cos \phi - u_2 \\ &= V_{\theta 1} \cos \phi - u_2 \\ &= (V_1 \cos \phi - u) \\ &= (V_1 - u) \cos \phi - u \end{aligned}$$

$$\text{Hydraulic efficiency} = \frac{R.P}{K.E/\text{sec}} = \frac{\rho \times a \times V_1 (1 + \cos \phi) (V_1 - u) \times u}{\frac{1}{2} \times \rho \times a \times V_1 \times V_1^2}$$

For maximum efficiency

$$= \frac{2 (1 + \cos \phi) (V_1 - u) \times u}{V_1^2}$$

$$\begin{aligned} \frac{d\eta}{du} = 0 &\Rightarrow \frac{d}{du} \left[ \frac{2 (1 + \cos \phi) (V_1 - u) \times u}{V_1^2} \right] \\ &\Rightarrow \boxed{u = \frac{V_1}{2}} \text{ (maximum efficiency)} \end{aligned}$$

Maximum hydraulic efficiency  $\eta_H = \frac{(1 + \cos \phi)}{2}$

(1) velocity of Jet at inlet  $V_1 = C_v \sqrt{2gH}$

(2) velocity of wheel  $(u) = \phi \sqrt{2gH}$

$\phi = \text{speed ratio} = \frac{\text{Blade speed}}{\text{theoretical}}$

(3) mean diameter  $u = \frac{\pi D N}{60}$

(4) Jet Ratio  $(m) = \frac{D}{d} \left( \frac{\text{pitch diameter}}{\text{dia of jet}} \right)$

(5) no. of buckets  $z = 15 + \frac{D}{2d}$

# Reaction Turbines

As we know the water available at the inlet is consisting of kinetic energy and pressure energy. The water enters to the runner a part of pressure energy gets converted to kinetic energy. The runner is completely enclosed in an air-tight casing. Runner is always full of water penstock

Main parts used are

(1) casing  $\rightarrow$  The water from penstock enters to the casing which is spiral in shape and it is always full of water and completely surrounds the runner.

(2) Guide Mechanism  $\rightarrow$  It is a stationary circular wheel all round the runner the guide vanes are fixed. This allows the water to strike the vanes fixed on the runner without shock.

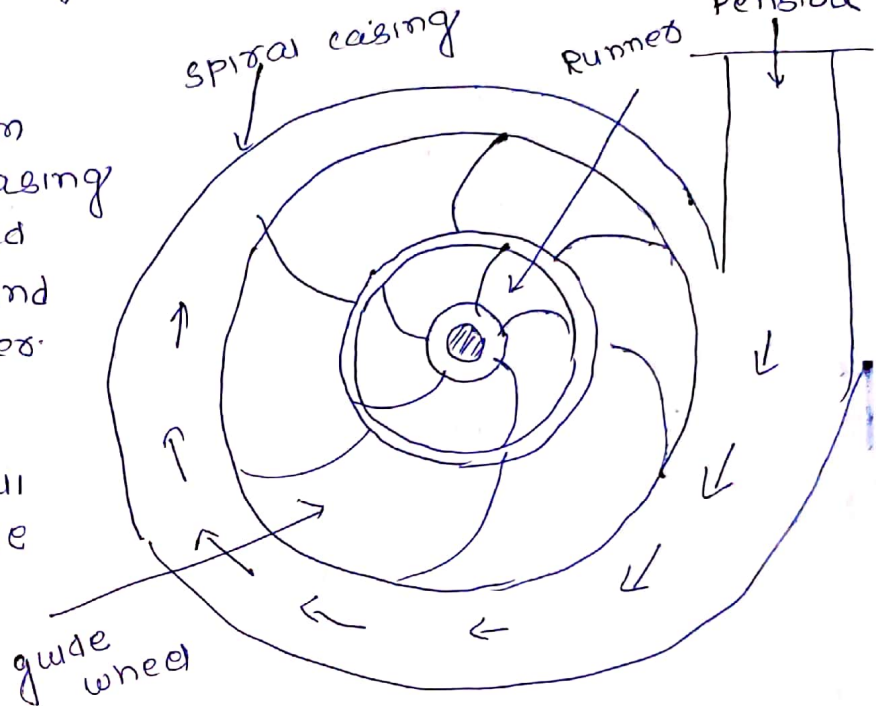
(3) Runner  $\rightarrow$  It is a circular wheel in which radial curved vanes are fixed and angled in such a way that water enters and leaves without shock.

(4) Draft tube  $\rightarrow$  The pressure at the exit of the runner of a reaction turbine is generally less than atmospheric pressure. The water can't be directly discharge so a tube of gradually increasing area is used for discharge of water.

## Velocity Diagram

The velocity diagram is same as impulse turbine.

$$\begin{aligned} R.P &= \frac{\rho \times a \times v_1 (v_{w1} u_1 + v_{w2} u_2)}{1000} \text{ kW} \\ &= \frac{\rho \times Q \times (v_{w1} u_1 + v_{w2} u_2)}{1000} \text{ kW} \end{aligned}$$



## Francis turbine

The inward flow reaction turbine having radial discharge at outlet is called Francis turbine. It is used for radial discharge  $\beta = 90^\circ$ ,  $v_{w2} = 0$  for medium head and high discharge.

$$\begin{aligned} R.P &= \frac{\rho \times a \times v_1 [v_{w1} u_1]}{1000} \text{ kW} \\ &= \frac{\rho \times Q [v_{w1} u_1]}{1000} \text{ kW} \end{aligned}$$

## Kaplan turbine

It is axial flow reaction turbine so the shaft is vertical. The lower end of the shaft is made larger. When the vanes are fixed in the hub and not adjustable then it is called propeller turbine and if the vanes are adjustable then it is called Kaplan turbine. It works on low head and high discharge.

$$\begin{aligned} Q &= A \times V \\ &= \frac{\pi}{4} \times (D_o^2 - D_b^2) \times v_{f1} \end{aligned}$$

$v_{f1}$  = Flow velocity at inlet

$D_b$  = diameter of hub

$D_o$  = outer diameter of runner

## Specific speed of turbine

It is defined as the speed of the turbine which is identical in shape, geometrical dimension, blade angle etc. with the actual turbine but it will develop unit power when working on unit head.

$$\begin{aligned} N_s &= \frac{N \sqrt{P}}{(H)^{5/4}} \\ &= \frac{N \sqrt{P}}{(Q)^{3/4}} \end{aligned}$$

Pelton  $\rightarrow$  8.5 to 30

Francis  $\rightarrow$  51 to 255

Kaplan  $\rightarrow$  255 to 860

## unit quantities

In order to predict the behaviour of a turbine working under varying condition of head, speed, output and gate opening the results are expressed in quantities.

(1) **unit speed**  $\rightarrow$  That speed of turbine where it works under unit head. ( $N_u$ )

$N$  = speed

$H$  = head

$u$  = tangential velocity

$$u \propto V$$

$$\propto \sqrt{H}$$

$$u = \frac{\pi DN}{60} \text{ so } u \propto N$$

$$\text{so } N \propto u$$

$$\Rightarrow N \propto \sqrt{H}$$

$$\Rightarrow N = K_1 \sqrt{H}$$

$$(u = v \times r)$$

$$(v = \sqrt{2gH})$$

If head of the turbine is unity then speed becomes unit speed

$$H = 1, N = N_u$$

$$N_u = K_1 \sqrt{1} = K_1$$

$$N = N_u \sqrt{H} \Rightarrow \boxed{N_u = \frac{N}{\sqrt{H}}}$$

(2) **unit discharge**  $\rightarrow$

$Q$  = discharge

$H$  = head

$a$  = Area

$$Q = A \times V$$

$$= A \times \sqrt{2gH}$$

$$Q \propto \sqrt{H}$$

$$\Rightarrow Q = K_2 \sqrt{H}$$

When  $H = 1$

$Q$  becomes  $Q_u$

$$Q_u = K_2 \sqrt{1} = K_2$$

Now putting the value

$$Q = Q_u \sqrt{H}$$

$$\Rightarrow \boxed{Q_u = \frac{Q}{\sqrt{H}}}$$

(3) **unit power**  $\rightarrow$

$H$  = head

$P$  = power

$Q$  = discharge

$$P = \rho \times g \times Q \times H$$

$$P \propto Q \times H$$

$$P \propto \sqrt{H} \times H$$

$$P \propto (H)^{3/2}$$

$$\Rightarrow P = K_3 (H)^{3/2}$$

When  $H = 1$   $P$  becomes  $P_u$

$$\boxed{P_u = \frac{P}{(H)^{3/2}}}$$



# Centrifugal Pump

The hydraulic machine which converts the Mechanical energy into hydraulic energy is called pump. When this conversion is possible with the help of centrifugal force acting on the fluid is called centrifugal pump. It also acts as inward radial flow reaction turbine. It works on the principle of forced vortex flow which means when a certain mass of liquid is rotated by an external torque the rise in pressure head obtained.

$$h = \frac{v^2}{2g} = \frac{(w \times r)^2}{2g} = \frac{w^2 r^2}{2g}$$

## Main parts of centrifugal pump

(1) **Impeller** → The rotating part of the centrifugal pump is called impeller consists of series of backward curved vanes. It is mounted on a shaft.

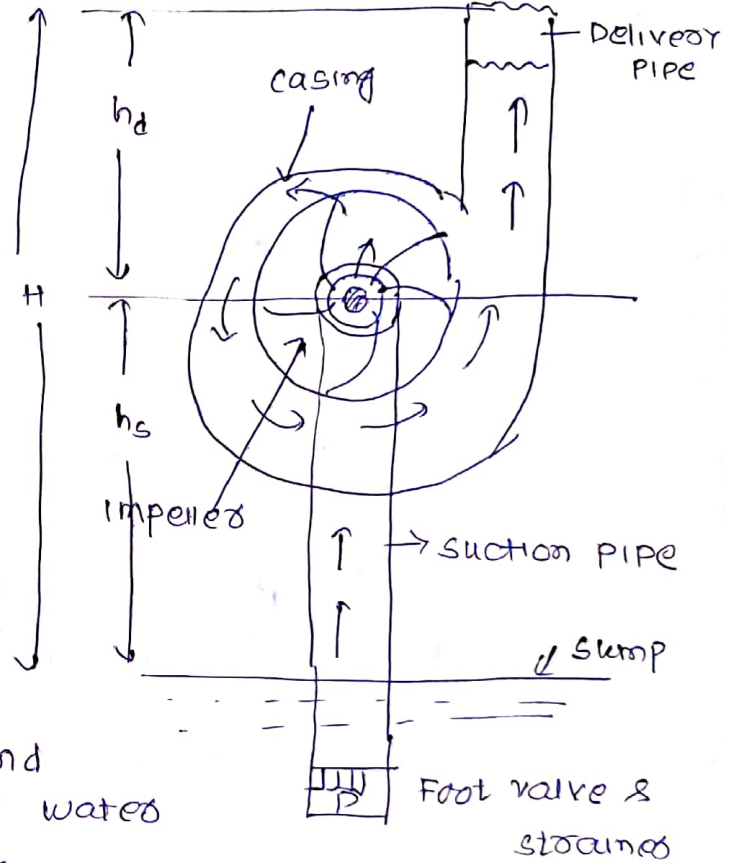
(2) **Casing** → The casing is an air-tight passage surrounds the impeller. It is a spiral type of gradually increasing cross-sectional area and designed in such a way that K.E of water discharged at the outlet of impeller.

(3) **Suction pipe with Foot valve and strainer** → A pipe whose one end is connected to the inlet of the pump and another end into the sump. The foot valve and strainer is also provided.

(4) **Delivery pipe** → A pipe whose one end is connected to the outlet of the pump and other end delivers water at a required height.

## Work done by centrifugal pump

Here the work is done by the impeller on water. and as we know the water enters the impeller in the radial direction so  $\alpha = 90^\circ$  and  $V_{w1} = 0$  it is due to obtain the best efficiency of the pump. As from earlier we know



$$R.P = \frac{\rho \times \alpha \times V_1 (v_{w1} u_1 + v_{w2} u_2)}{1000} \text{ kW}$$

$$\text{AS } \alpha = 90^\circ$$

$$v_{w1} = 0$$

$$= \frac{\rho \times \alpha \times V_1 (v_{w2} u_2)}{1000} \text{ kW}$$

$$\text{The discharge (Q)} = A \times V = \pi \times D_1 \times B_1 \times v_{f1}$$

$$= \pi \times D_2 \times B_2 \times v_{f2}$$

$B_1, B_2$  width of impeller at inlet and outlet

### Different types of head of a centrifugal pump

(1) suction head  $\rightarrow$  vertical height of the centre line of the pump above the water surface that is to be lifted.

(2) delivery head  $\rightarrow$  vertical distance from centre line of the pump and the water surface in tank water to be delivered.

(3) static head  $\rightarrow$  sum of (suction + delivery) head

(4) Manometric head  $\rightarrow$  It is defined as the head against which the centrifugal pump has to work.

$$H_m = h_s + h_d + \frac{V_d^2}{2g} + h_{fs1} + h_{fs2}$$

Frictional head loss in inlet and outlet

### Efficiencies of centrifugal pump

(a) Manometric Efficiency  $\rightarrow \eta_{\text{man}} = \frac{\text{Manometric head}}{\text{Head imparted by impeller}}$

$$\eta_{\text{mano}} = \frac{h_m}{\frac{v_{w2} u_2}{g}} = \frac{g \times h_{\text{mano}}}{v_{w2} u_2}$$

(b) Mechanical efficiency  $\rightarrow \eta_{\text{mech}} = \frac{\text{Power at the impeller}}{\text{Power at the shaft}}$

(c) overall efficiency  $\rightarrow \eta_{\text{overall}} = \frac{\text{Power output of pump}}{\text{Inlet of pump}}$

$$= \eta_{\text{mano}} \times \eta_{\text{mech}}$$

$$= \frac{W \times H_m}{1000} \text{ shaft power}$$

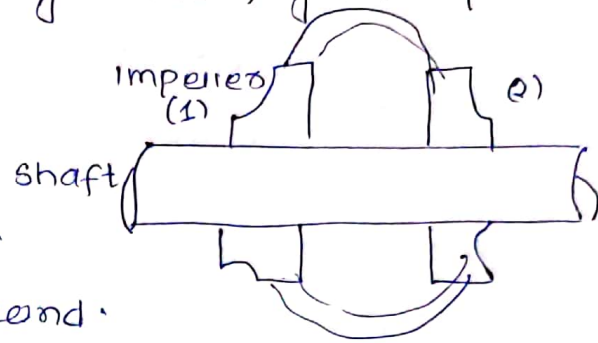


## Multistage centrifugal pump

If a centrifugal pump consists of two or more impellers then the pump is called multistage centrifugal pump.

(1) To obtain a high head  $\rightarrow$

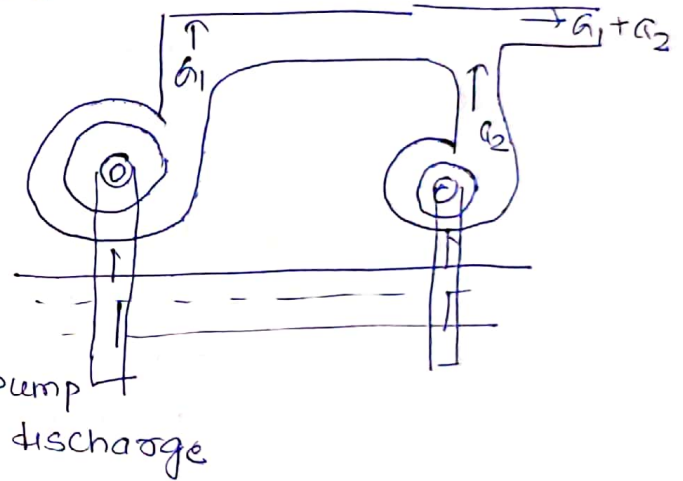
To obtain high head the impellers are connected in series so the discharge of 1st impeller enters to the second.



Total head =  $n \times H_m \rightarrow$  head developed by each impeller  
 $\downarrow$   
 no. of impellers

(2) To obtain high discharge  $\rightarrow$

The pumps are connected in parallel, each pump lift water from the common sump and discharge water to a common pipe.



Total discharge =  $n \times Q \rightarrow$  one pump  
 $\downarrow$   
 no. of pump

## Specific speed of centrifugal pump

It is defined as the speed of geometrically similar pump which would deliver one cubic metre of liquid per second against head of one metre.

$$N_s = \frac{N \sqrt{Q}}{(H_m)^{3/4}}$$

$H_m =$  manometric head

## Model testing of centrifugal pump

(1) Specific speed of model = specific speed of prototype

$$\left[ \frac{N \sqrt{Q}}{(H_m)^{3/4}} \right]_{\text{Model}} = \left[ \frac{N \sqrt{Q}}{(H_m)^{3/4}} \right]_{\text{Prototype}}$$

(2) tangential velocity  $\rightarrow u = \frac{\pi D N}{60} \quad \therefore v = \sqrt{2gh} = \alpha \sqrt{H_m}$

$$u \propto \sqrt{H_m}$$

$$\sqrt{H_m} \propto D \times N$$

$$\frac{\sqrt{H_m}}{D \times N} = \text{constant}$$

$$\begin{aligned}
 (3) \text{ Discharge } Q &= \pi \times D \times B \times v_{f1} \\
 &= \pi \times D^2 \times v_f \\
 &= \pi \times D^2 \times D \times N \\
 &= \pi \times D^3 \times N
 \end{aligned}$$

$$\frac{Q}{D^3 N} = \text{constant}$$

$$(4) \text{ Power } P = \rho \times g \times Q \times H_m$$

$$P \propto Q \propto H_m$$

$$P \propto D^3 \times N \times D^2 N^2$$

$$P \propto D^5 N^3$$

$$\frac{P}{D^5 N^3} = \text{constant}$$

### Priming of centrifugal pump

It is the operation in which the suction pipe, casing of the pump and a portion of delivery pipe upto delivery valve is completely filled up from outside source with the liquid to be raised by the pump before starting. Thus air from the parts is removed.

### Cavitation

It is defined as the phenomenon of formation of water or vapour bubbles of a flowing liquid in a region where the pressure of liquid falls below vapour pressure and collapsing of the vapour bubbles in a region of high pressure. Due to the sudden collapsing of the bubbles noise and vibration is produced. Efficiency of the pump decreases, cavities are formed.

$$\sigma = \frac{H_b - H_s}{H} = \frac{(H_{atm} - H_v) - H_s}{H}$$

Thoma's cavitation Factor

$H_b$  = Barometric pressure head

$H_{atm}$  = atmospheric " "

$H_v$  = vapour pressure "

$H$  = Total head of pump

$H_s$  = suction pressure head

(\* ) Both Reaction turbine and centrifugal pumps are subjected to cavitation.

## Reciprocating Pump

If the Mechanical energy gets converted to hydraulic energy by sucking the liquid into a cylinder in which a piston which gives Reciprocating motion is called Reciprocating Pump.

## Parts of Reciprocating Pump

It is a single acting Reciprocating Pump consists of this arrangement. The suction and delivery valve are one way valve which allows the water to flow in one direction.

When the crank starts rotating the piston moves to and fro. When the crank moves from the point A to C  $\theta = 180^\circ$  the piston moves towards right of the cylinder so it creates a partial vacuum but not greater than atmospheric

pressure so the liquid is forced into the suction pipe from the sump. liquid opens suction valve and enters the cylinder.

When the crank is rotated from C to A  $\theta = 360^\circ$  from extreme right it starts towards left in the cylinder. The movement towards left increases the pressure which is greater than atmospheric pressure hence suction valve close and delivery valve opens and liquid is forced through the delivery pipe.

## Discharge and work done by Reciprocating Pump

D = diameter of cylinder

r = radius of crank

$$A = \frac{\pi}{4} \times D^2$$

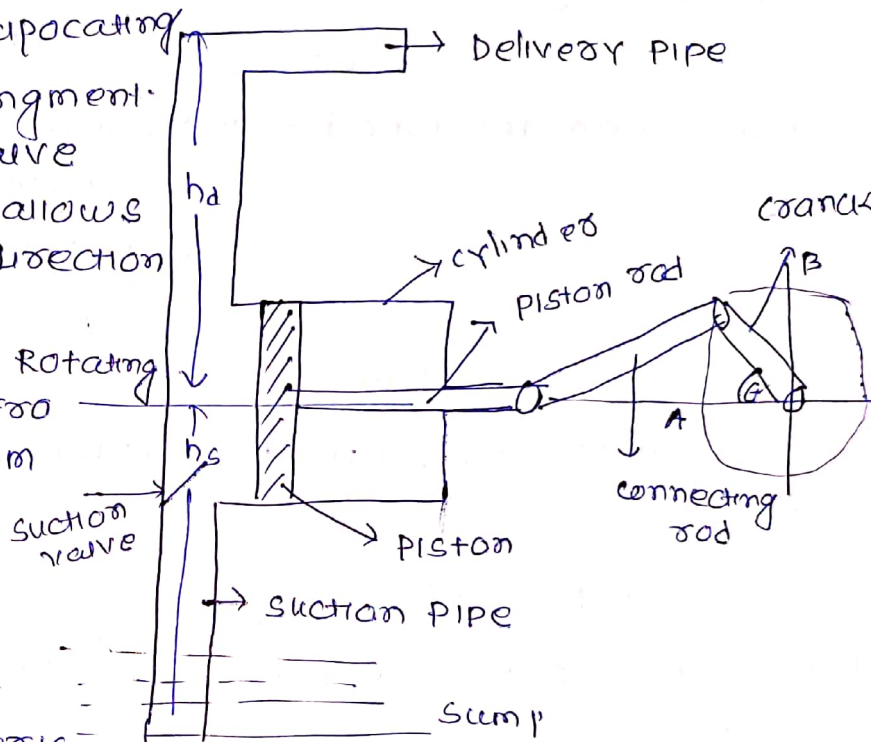
N = r.p.m of crank

L = length of stroke = 2r

Volume of water delivered =  $A \times L$

no. of Revolution per second =  $\frac{N}{60}$

discharge of pump per second =  $A \times L \times \frac{N}{60}$



$$\begin{aligned} \text{Work done} &= \text{weight of water lifted} \times (\text{Total height through which lifted}) \\ &= W \times (h_s + h_d) \end{aligned}$$

$$\text{weight of water lifted} = \rho \times g \times Q = \frac{\rho \times g \times A \times L \times N}{60}$$

$$\text{Work done} = \frac{\rho \times g \times A \times L \times N}{60} \times (h_s + h_d)$$

$$\begin{aligned} \text{Total power developed} &= \frac{\text{Work done per second}}{1000} \\ &= \frac{\rho \times g \times A \times L \times N \times (h_s + h_d)}{60,000} \text{ kW} \end{aligned}$$

### Slip of Reciprocating Pump

It is defined as the discharge (theoretical) - actual discharge of the pump.

$$\text{Slip} = Q_{th} - Q_{actual}$$

But most time it is expressed in percentage

$$= \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100$$

$$= \left(1 - \frac{Q_{act}}{Q_{th}}\right) \times 100 = (1 - C_d) \times 100$$

### Questions on Turbines

~~\*\*\*~~

(1) The specific speed of the turbine is given by

$$(a) N_s = \frac{N\sqrt{P}}{(H)^{3/4}} \quad (b) N_s = \frac{N\sqrt{P}}{(H)^{5/4}} \quad (c) N_s = \frac{N\sqrt{Q}}{(H)^{3/4}} \quad (d) N_s = \frac{N \cdot P^2}{(H)^{2/3}}$$

(2) The discharge through Kaplan turbine is given by

$$(a) Q = \pi \times D \times B \times v_f \quad (b) Q = \frac{\pi}{4} \times d^2 \times \sqrt{2gh} \quad (c) \frac{\pi}{4} \times [D_o^2 - D_b^2] \times v_f \quad (d) \text{NONE}$$

(3) unit speed is the speed of the turbine when working at

(a) unit head (b) unit head & discharge (c) unit head & unit power

(4) Francis turbine is a (1) Impulse turbine (2) Reaction turbine

(3) axial flow turbine (d) Reaction Radial flow turbine

## Question on pumps

(1) cavitation can take place in case of

- (1) Pelton wheel (2) Francis turbine (3) Centrifugal Pump (4) Reciprocating Pump

(2) cavitation occurs when

(1) Pressure of liquid = vapour pressure of the liquid

(2) " " Fluid is less than vapour pressure of liquid

(3) " " Fluid is above the " " " liquid

(4) NONE

(3) centrifugal pump works on the principle of

(1) Laminar flow (2) Free vortex flow (3) Forced vortex flow

(4) If the specific speed of a turbine is more than 300 then the type of turbine is

(1) Pelton (2) Francis (3) Kaplan (4) All

(5) The flow of water leaving the impeller is

(1) forced vortex flow (2) free vortex flow (3) laminar

(6) For low head and high discharge the suitable turbine is

(1) Pelton wheel (2) Francis (3) Kaplan (4) All

## Problems

(1) A Pelton wheel turbine has a mean bucket speed 10 m/s with a jet of water flowing at the rate of 700 litres/sec under a head of 30 m. The angle of jet deflection =  $160^\circ$  calculate the power given by water to the runner and hydraulic efficiency assume

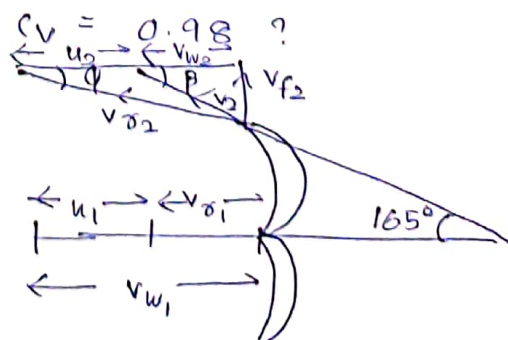
$$u = 10 \text{ m/s} = u_1 = u_2$$

$$Q = 700 \text{ l/sec}$$

$$H = 30 \text{ m}$$

$$\phi = 180^\circ - 160^\circ = 20^\circ$$

$$C_v = 0.98$$



$$\text{inlet velocity } (v_1) = C_v \sqrt{2gH}$$

$$= 0.98 \sqrt{2 \times 9.81 \times 30} = 23.77 \text{ m/sec}$$

$$V_{\alpha_1} = v_1 - u_1 = 23.57 - 10 = 13.77 \text{ m/sec}$$

$$\text{For Pelton } V_{\alpha_1} = V_{\alpha_2} = 13.77 \text{ m/sec}$$

$$\begin{aligned} \text{at outlet } V_{w_2} &= V_{\alpha_2} \cos \alpha - u_2 = 13.77 \cos 20^\circ - 10 \\ &= 2.94 \text{ m/s} \end{aligned}$$

$$\begin{aligned} \text{Runner power} &= \frac{\rho \times a \times v_1 \times (V_{w_1} u_1 + V_{w_2} u_2)}{1000} \text{ Kw} \\ &= \frac{1000 \times 0.7 \times 23.77 [23.77 \times 10 + 2.94 \times 10]}{1000} \text{ Kw} \\ &= 186.97 \text{ Kw} \end{aligned}$$

$$\begin{aligned} \text{hydraulic efficiency} &= \frac{R.P.}{W.P.} = \frac{186.97}{\rho \times g \times a \times H} = \frac{186.97}{1000 \times 9.81 \times 700 \times 30} \\ &= 94.54 \% \end{aligned}$$

(2) A turbine to operate under a head of 25 m at 200 r.p.m. The discharge is 9 m<sup>3</sup>/sec and Find water power and specific speed of the turbine?

$$\begin{aligned} \text{water power} &= \rho \times g \times a \times H \\ &= 1000 \times 9.81 \times 9 \times 25 \\ &= 1986.5 \text{ Kw} \end{aligned}$$

$$\begin{aligned} \text{specific speed } N_s &= \frac{N \sqrt{P}}{(H)^{5/4}} = \frac{200 \times \sqrt{1986.5}}{(25)^{5/4}} = 159.46 \text{ r.p.m} \\ &\quad \text{---X---X---X} \end{aligned}$$