LECTURE NOTES

ON
THEORY OF MACHINES

4^{TH} SEMESTER

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Theory of Machine

Chapters include →

(1) concept of simple Machine
(2) Four bar linkage & link motion
(3) Friction in pivot and collar
(4) Clutches
(5) Fluctuation of Energy
(6) Flywheel
(7) Governor
(8) Cams and Follower

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Concept of Simple Machine

a Simple machine may be defined as a device which enables us to do some useful work at some point or to overcome Resistance when an effort or force is applied to it.

Compound machine

It is consisting of a no. of simple machines which enable us to do some useful work at a better speed.

Lifting Machine

It is a device which enable us to lift a heavy load (P) by applying a small effort.

Mechanical Advantage

It is defined as the Ratio between weight lifted to the effort applied and expressed in numbers. $M.A = \frac{W}{P}$

Input of a Machine

The work done on the machine: in lifting machine = effort x distance

Output of a Machine

The output of a machine is the actual work done by the machine in lifting machine = weight lifted x distance

Effciency of a Machine

It is the Ratio of output to the input of a Machine.

Ideal Machine

If the effciency of a machine is 100% that means input is equal to output.

Velocity Ratio

$$\frac{\text{Distance moved by effort}}{\text{Distance moved by the load}} = \frac{Y}{X}$$

Relationship between all Parameters

W = load lifted

P = effort Required

Y = distance moved by effort

X = distance moved by load

$$M.A = \frac{W}{P} \quad \text{and} \quad V.R = \frac{Y}{X}$$

Input of a machine = P × Y

Output of a Machine = W × x

$$\eta = \frac{output}{input} = \frac{W \times x}{P \times Y} = \frac{W/P}{Y/x} = \frac{M.A}{V.R}$$

Reversiblity of a Machine

Sometimes a machine is capable of doing some work in the Reversed direction after the effort is Removed. Such machines are called Reversible machine. For a Reversible machine it's effciency should be more than 50%.

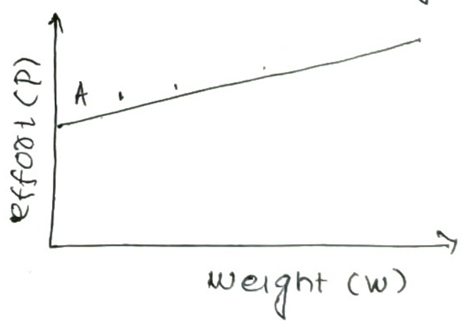
non-Reversible Machine

Sometimes a machine is not capable of doing any work in the Reversed direction even if the effort is Removed. here the effciency should be less than 50%. (self-locking)

Law of Machine

It is defined as Relationship between the effort applied and the load lifted. Thus For any machine we plot the graph we are getting a straight line.

$$\boxed{P = mW + C}$$



P = effort applied

m = constant (μ of Friction)

W = Load Lifted

C = constant of Machine Friction.

maximum Mechanical Advantage $= \frac{W}{P} = \frac{W}{mW + C} = \frac{1}{m + \frac{C}{W}} = \frac{1}{m}$ (neglect $\frac{C}{W}$)

maximum effciency of a lifting machine

$$\eta = \frac{M.A}{V.R} = \frac{\frac{W}{P}}{V.R} = \frac{W}{P \times V.R} = \frac{W}{(mW + C) \times V.R} = \frac{1}{(m + \frac{C}{W}) \times V.R} = \frac{1}{m \times V.R}$$

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Four bar linkage and Link motion

Each part of the machine which moves relative to some other part is known as kinematic link or link or element. It is must be a Resistant body so that it can transmite Required amount of force.

Types of Link

(1) Rigid link → A Rigid link is one which does not undergoes any deformation while transmitting motion. In actual practice there is no Rigid link.

(2) Flexible link → which partially deformed while transmitting motion.

(3) Fluid link → motion is transmitted through Fluid pressure.

Structure

It is the assembles of a number of Resistant bodies having no Relative motion between them.

Kinematic chain

When the kinematic pair are coupled in such a way that the last link is Joined to the First link to transmite definite motion is called kinematic chain

Kinematic Pair

The two link or elements of a machine when in contact with each other they should form a pair. If the relative motion between them is completely or sucessfully constrainted is called kinematic pair

Types of constrained motion

(1) completely constrained motion → when the motion between pair is limited to a definite direction irrespective of the applied Force.

(2) incompletely constrained motion → when the motion between a Pair takes place in more than one direction.

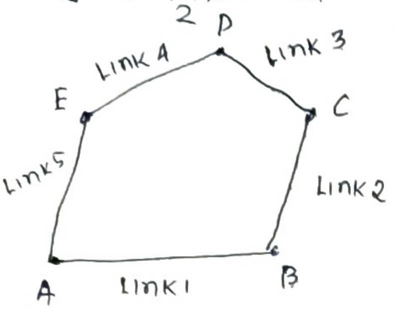
(3) Successfully constrained motion → when the motion between the element forming a pair such that there motion is completely constrained to any of the direction.

kinematic chain linkage

If each link is assumed to be form two pair of adjecent link then the Relationship between the pair (P) and Link (L) is

$L = 2P - 4 \quad \dots (1)$

$J = \frac{3}{2} \times L - 2 \quad \dots (ii)$



$L = 5, P = 5, J = 5$

$L = 2P - 4$

$5 = 2 \times 5 - 4 = 6$

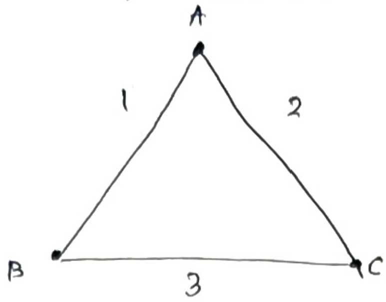
L.H.S < R.H.S

$J = \frac{3}{2} \times 5 - 2$

$5 = 5.5$

L.H.S < R.H.S

not a kinematic chain called unconstrained motion



(1) $L = 3$

$J = 3$

$P = 3$

$L = 2P - 4 = 2 \times 3 - 4$

$= 6 - 4$

$= 2$

L.H.S > R.H.S

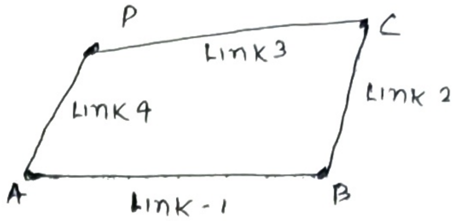
(2) $J = \frac{3}{2} \times L - 2$

$3 = \frac{3}{2} \times 3 - 2$

$= 2.5$

L.H.S > R.H.S

Since it does not satisfy it is not a kinematic chain. and locked chain



$L = 4, J = 4, P = 4$

$L = 2 \times 4 - 4 = 4$

L.H.S = R.H.S

$J = \frac{3}{2} \times 4 - 2 = 4$

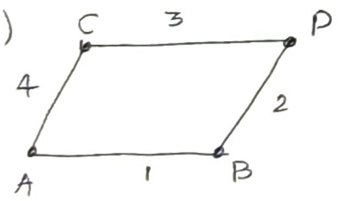
4

L.H.S = R.H.S

It is a kinematic chain compltely constrined motion.

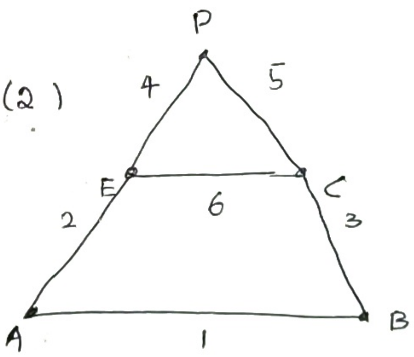
Types of Joint in a chain

(1)



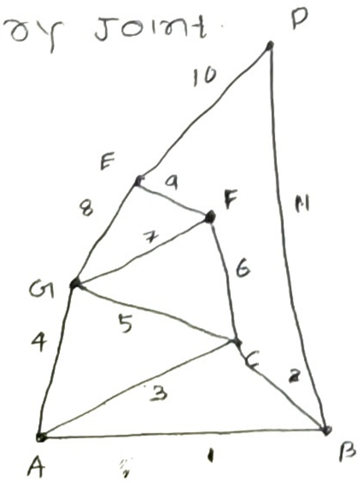
when two links are joined in same connection is called binary joint.

(2)



A, B, D are binary joint and E and C are ternary joint.

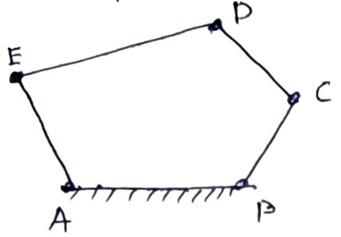
(3)



when four links are joined in the same connection is called quaternary joint.

C and G are quarternary joint.

examples →



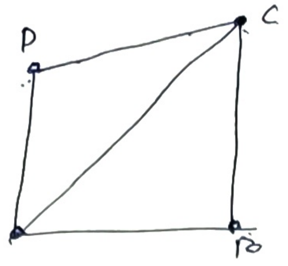
here $L = 5$

$J = 5$

$n = 3(5-1) - 2\times5 - 0$

$= 3\times4 - 10 = 2$

(2)



$L = 5$

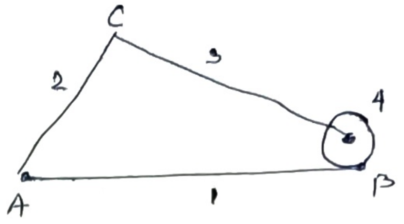
$J = 6$

$n = 3(5-1) - 2\times6$

$= 3\times4 - 12$

$= 0$

(3)



$L = 4$

$J = 3$

$h = 1$

$n = 3(4-1) - 2\times3 - 1$

$= 9 - 6 - 1$

$= 2$

Grubler's criterion for Plane mechanism

It applies to mechanism having single degree of freedom that means $n = 1$ and higher pair is not there.

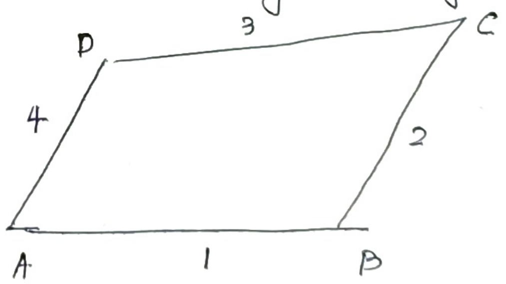
$n = 3(L-1) - 2J - h$ $\Rightarrow$ $3L - 2J - 4 = 0$

$1 = 3(L-1) - 2J - 0$ $\Rightarrow$ $\boxed{3L - 2J = 4}$

Inversion of Mechanism

When one link is Fixed in a Kinematic chain it is called as Mechanism. so the process of obtaining different Mechanism by fixing different link of a Kinematic chain is called inversion of Mechanism. The most important Kinematic chain are those which consists of Four lower pair each pair having sliding or turning pair.

(1) Four bar chain



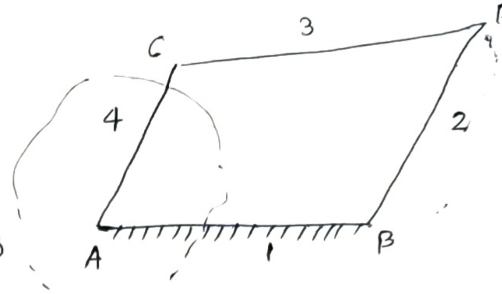
The simplest or basic Mechanism is called the Four bar chain. consists of Four link each Pair must be turning or sliding pair. The Four links have dif^f Length

According to Grashof's Law for a Four bar Mechanism the sum of shortest and Longest link should not be greater than Remaining two Links.

$$\boxed{L + S \leq P + q}$$

While designing a Four bar chain one of the link must do complete Revolution to Form a Sucessful Mechanism.

In particular the shortest link will make a complete Revolution. such a link is called cranck or driver.



The link BC does partial Rotation so is called Rocker.

AC = cranck

BD = Lever / Rocker

The link which connects cranck and lever is CD is called connecting Rod. The Fixed link is called Frame.

The inversion of Four bar chain are →

(1) Beam engine (2) coupling Rod locomotive (3) Double lever Mechanis

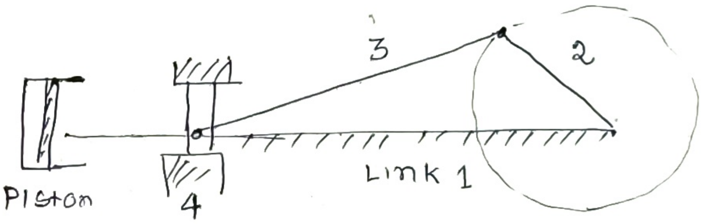
(2) single slider cranck chain

It is the modification of the Four bar chain consists of one sliding pair and three turning pair. It is usually Found in Recipocating steam engine.

Link 4-1 (sliding pair)

Link (1-2) (2-3) (3-4)

Turning pair



Link 1 = Frame, Link 2 = cranck, Link 3 = connecting rod, Link 4 = cross-head

inversion of single slider cranck →

(1) Pendulump pump / Bull engine
(2) oscillating cylinder engine
(3) Rotary internal combustion
(4) cranck and slotted q.R.R Mech
(5) whitworth q.R.R Mechanism

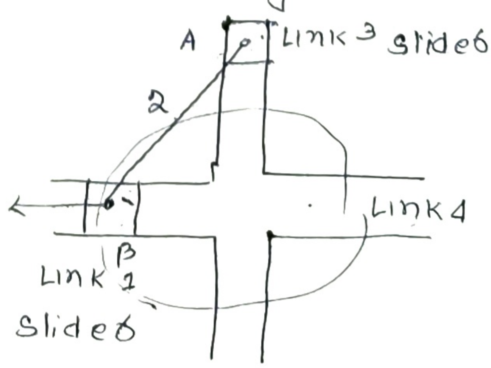
(2) Double slider cranck chain

Kinematic chain which consisting of two turning pair & two sliding pair is known as Double slider cranck chain.

Link 1 and 2 → Turning pair
Link 3 and 4 → sliding pair
Link 2 and Link 3 → Turning pair
Link 1 and Link 4 → sliding pair

inversion of Double cranck Mechanism →

(1) Eliptical trammel
(2) scotch Yoke Mechanism
(3) Oldham's coupling



Classification of kinematic pair

According to the type of Relative motion →

(a) Sliding pair → when two elements of a pair having sliding motion Relative to one another.

(b) Turning pair → when two elements of a pair connected in such a way that one having turn or Revolve about another link.

(c) Rolling pair → when two elements connected in such a way that one Rolls over another fixed link.

According to the type of contact →

(a) Lower pair → when two elements of a pair having surface contact with each other. (slides over another)

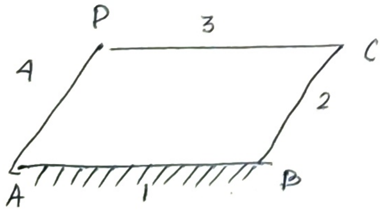
(b) Higher pair → when two elements of a pair having line or Point contact when Relative motion takes place (partially turning or sliding)

Mechanism

When one of the links of a kinematic chain is fixed, the chain is called Mechanism. It is used for transmitting motion. A mechanism with four links is called simple mechanism and more than four link is compound Mechanism. When the mechanism is used to transmite power is becomes a Machine.

Degree of Freedom

It is also called movability of the mechanism. It is defined as the no. of input parameter which must be independtly controlled to use it for a useful work.



Mechanism

Kutzbach criterion for degree of freedom

$$n = 3(L-1) - 2J - h$$

when there is no higher pair

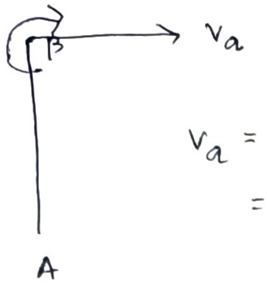
$$n = 3(L-1) - 2J$$

L = no. of link

J = no. of Joints

Concept on velocity and Accelaration

Velocity at any point on a link is given by the Analysis

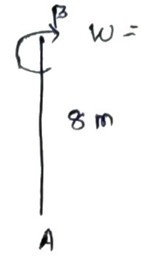


$V_a = r \times w$

$= r$ = length of AB

w = angular Rotation

ex



$V_B = w \times r$

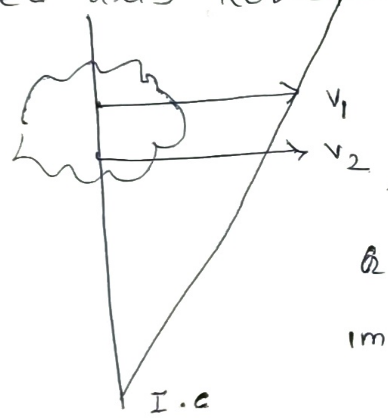
$= 8 \times 8$

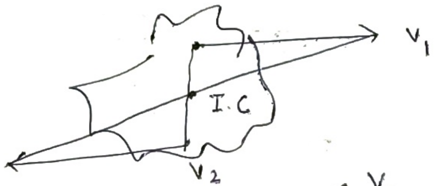
$= 64$ m/sec

velocity calculation can also be done by instantaneous centre Method.

Instantenous Centre Method

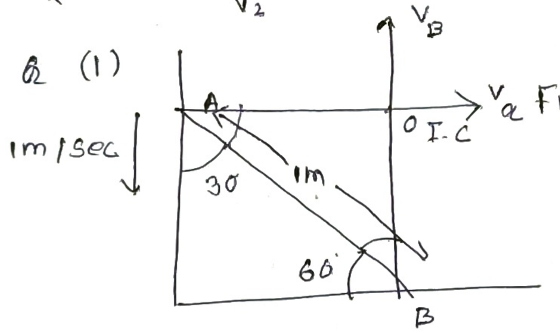
It is defined as the point with Respect to which a body undergoes general Plane motion and assumed to be undergoes Fixed axis Rotation.





The velocity at the instantenous centre is always zero.

Q (1)



Find the angular velocity of the Rod at this instant ?

$V_a = r \times w$

$= w \times r_{OA}$

$w = \frac{V_a}{r_{OA}} = \frac{1}{1/2} = 2$ Rad/sec

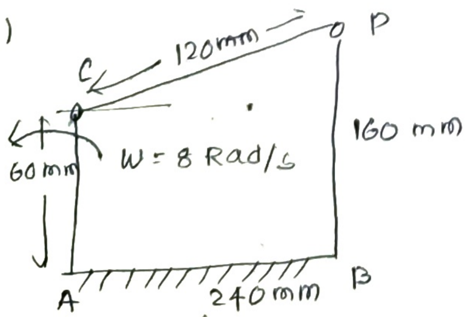
$V_B = r \times w$

$= r_{OB} \times w$

$= \sin 60^\circ \times 1 \times 2$

$= 1.732$ rad/sec

Q (2)



Find the Angular velocity of BD ?

Ans → $V_{BD} = r_{BD} \times w$

As the linear velocity is same

$w_1 \times r_{AC} = w_2 \times r_{BD}$

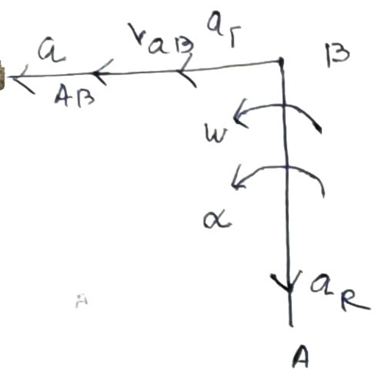
$8 \times 60 = w_2 \times 160$

$w_2 = \frac{480}{160} = 3$ rad/sec

no. of instantaneous centres in a Mechanism is given by

$$N = \frac{n(n-1)}{2} \quad \text{where } n = \text{no. of link}$$

Accelaration in a Mechanism



when accelaration is taken ito account we have to consider both radial as well as tangential component

$$a_{Radial} = w^2 \times r = w^2 \times r_{AB}$$

$$a_{tangential} = r \times \alpha = \alpha \times r_{AB}$$

Resultant accelaration $a_R = \sqrt{(a_t)^2 + (a_R)^2}$

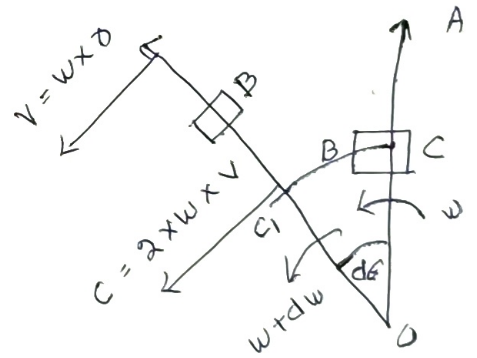
Coriolis Component of Accelaration

when a point on the link is sliding along another Rotating link then coriolis component of accelaration must be calculated.

which is given by $C = 2 \times w \times V$

w = angular velocity of link

V = linear velocity of link



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Questions

(1) The motion of piston in the cylinder in a steam engine is an example of

(a) completely constrained (b) incompletely constrained (c) sucessfully constrained

(2) A Kinematic chain is known as Mechanism when

(a) one link is fixed (b) Two link is fixed (c) Three link fixed (d) None

(3) The mechanism forms a structure when the degree of freedom is equal to

(a) zero (b) one (c) Two (d) Three

(4) In a four bar chain we have →

(1) one Turning pair & Two sliding (b) one Turning & Three sliding

(5) If in a Mechanism there are total 6 links How many instaneous centre can be Found ?

(a) 20 (b) 30 (c) ✓ 15 (d) 60

(6) In a Mechanism the Fixed instaneous centres are those

(a) ✓ Remain in the same plane For all configuration

(b) varying with the configuration of the Mechanism

(c) moves when the mechanism moves

(d) All

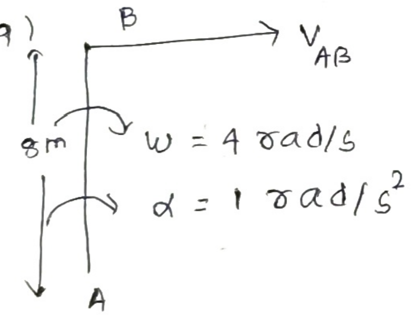
(7) The direct of linear velocity of any point on a link w.r.to another

(a) parallel to the link (b) at 45° to the link Joining (c) ✓ perpendicular to the link

(8) The magnitude of linear velocity of a link of length 5 m and moves with angular velocity of 4 rad/s Find the velocity ?

(a) 25 m/s (b) ✓ 20 m/s (c) 35 m/s (D) 0 m/sec

(9)



Find the velocity V_{AB} ? Ans $V_{AB} = 8 \times 4 = 32$

Find Radial acceration ? $a_R = 4^2 \times 8 = 128$

Find tangential acceration ? $a_T = 8 \times 1 = 8$

Find Resultant acceration ? $a_R = \sqrt{(8)^2 + (128)^2}$

(10) A link AB of length 5 m moves with angular velocity 6 rad/s Find the linear velocity and if the link has a slider whose velocity is 5 m/sec Find the corilios acceration of the link ?

linear velocity $V_{AB} = w \times r = 6 \times 5 = 30$ m/sec

$C_c = 2 \times w \times V = 2 \times 6 \times 5 = 60$ rad/sec

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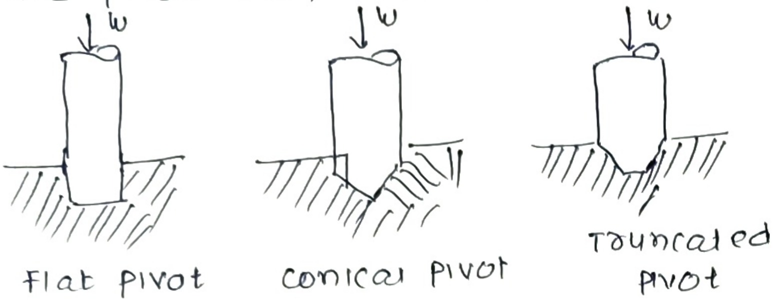
Friction in Pivot and collar

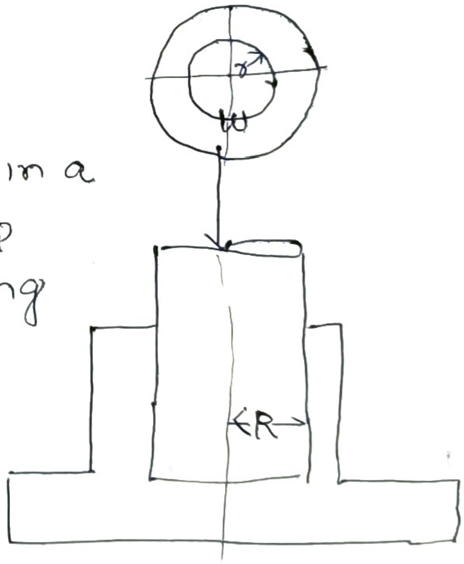
The bearing surface which is placed at the end of the shaft to carry out the axail thrust is called as pivot.
The pivot may have Flat, conical or truncate surface.



while studying about the pivot and collar we have assume that for a new bearing the pressure is uniformly distibuted which is called the condition of uniform pressure.

while when the bearing becomes old pressure is not uniformly distributed velocity due to the velocity of Rubbing surface some wear occurs. Due to this we have another conditions which is called uniform wear.

Flat pivot Bearing



when a vertical shaft Rotates in a Flat pivot Bearing known as foot step bearing. The sliding Friction will be along the surface of contact.

W = axail load

R = Radius of bearing

p = intensity of pressure

μ = co-effcient of Friction

1) uniform pressure →

As we know pressure is uniformly distributed we have

$$p = \frac{F}{A} = \frac{W}{\pi R^2}$$

consider a ring of radius r and thickness dr of bearing Area

$$A = 2\pi r \cdot dr$$

$$\text{Load} = p \times A = p \times 2\pi r \cdot dr$$

↓

$$dW = p \times 2\pi r \cdot dr$$

Frictional Resistance $F_r = \mu \times dw$

$= \mu \times p \times 2\pi r dr$

$= 2\pi \mu \times p \times r \times dr$

Frictional Torque transmitted $T_r = F_r \times r$

$= 2\pi\mu \times p \times r \times dr \times r$

$= 2\pi\mu \times p \times r^2 \times dr$

To findout for total bearing surface Area we have to integrate 0 to R

$$T = \int_0^R 2\pi\mu \times p \times r^2 dr = 2\pi\mu \times p \times \int_0^R r^2 dr = 2\pi\mu p \times \left[\frac{r^3}{3}\right]_0^R$$

$$= \frac{2}{3}\pi \times \mu \times p \times R^3 = \frac{2}{3} \times \cancel{\pi} \times \mu \times \frac{W}{\cancel{\pi} \cancel{R^2}} \times R^{\cancel{3}}$$

$$T = \frac{2}{3} \times \mu \times W \times R$$

If the shaft rotates at ω rad/sec

$$P = T \times \omega = T \times \frac{2\pi N}{60}$$

2) considering uniform wear →

As we know wear $\propto P \propto V$ that means if the Rubbing velocity increases wear Rate increses. That means as we move towards R the wear Rate increases. To considered as uniform wear $p \times r$ = constant $\Rightarrow p = \frac{C}{r}$

$p = \frac{F}{A} \Rightarrow F = \text{Load} = p \times A$

$$dw = \frac{C}{\cancel{r}} \times 2\pi \cancel{r} . dr$$

Total load $= W = \int_0^R 2\pi C dr = 2\pi C \times R \Rightarrow C = \frac{W}{2\pi R}$

~~Total Torque (T_r) = $\int_0^R 2\pi\mu$~~

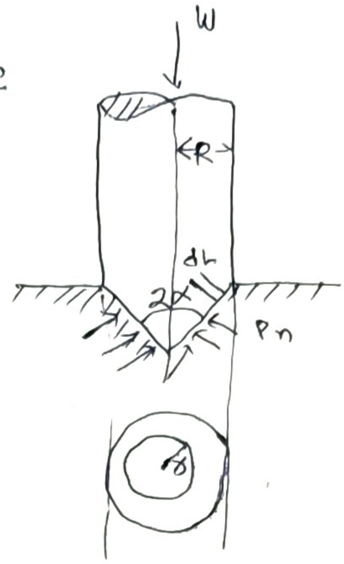
Frictional Resistance $(F_r) = \mu \times dw = \mu \times 2\pi C \times R$

Frictional Torque acting $T_r = F_r \times r = 2\pi\mu \times p \times r^2 \times dr$

$$= 2\pi\mu \times \frac{C}{\cancel{r}} \times r^{\cancel{2}} dr = 2\pi\mu \times C \times r \times dr$$

Total Frictional Torque $(T) = \int_0^R 2\pi\mu \times C \times r \times dr = \frac{1}{2} \times \mu \times W \times R$

Conical Pivot Bearing



P_n = intensity of pressure Normal to the cone

α = semi angle of cone

μ = co-efficient of Friction

R = Radius of the shaft.

considering a small Ring of radius r and thickness dr

dL = Length of Ring along the cone

$$dL = dr \operatorname{cosec}\alpha$$

$$A = 2\pi r . dL = 2\pi r . dr . \operatorname{cosec}\alpha$$

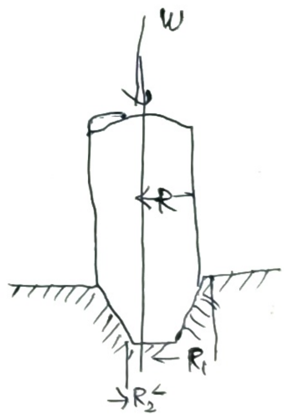
i) uniform pressure →

$$\text{Total (Torque)} = \frac{2}{3} \times \mu \times W \times R \times \operatorname{cosec}\alpha$$

ii) uniform wear →

$$\text{Total (Torque)} = \frac{1}{2} \times \mu \times W \times R \times \operatorname{cosec}\alpha$$

Trapezoidal / Truncated conical Pivot Bearing



If the pivot bearing is not conical but a Frustrum of a cone with radius r_1 and r_2 be the external & internal Radius

$$A = \pi [(r_1)^2 - (r_2)^2]$$

$$P_n = \frac{W}{A} = \frac{W}{\pi [(r_1)^2 - (r_2)^2]} .$$

i) uniform pressure →

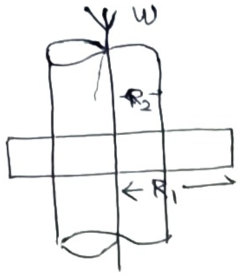
$$T = \frac{2}{3} \times \mu \times W \times \operatorname{cosec}\alpha \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

ii) uniform wear →

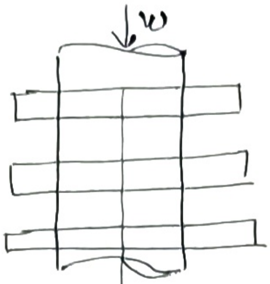
$$T = \frac{1}{2} \times \mu \times W \times \operatorname{cosec}\alpha \, (r_1 + r_2)$$

Flat collar Bearing

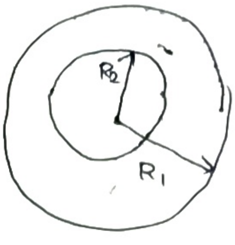
collar Bearing are used to take the axail thrust of the rotateing shaft. There may be a single collar or multiple collar bearing. The collar bearings are also called thrust bearing.



single collar



multiple collar



r_1 = external Radius of the collar r_2 = internal Radius of collar

Area of bearing surface $A = \pi[(r_1)^2 - (r_2)^2]$

$$P = \frac{W}{A} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]}$$

1) uniform pressure →

$$P = \frac{W}{A} = \frac{W}{\pi[R_1^2 - R_2^2]}$$

Area of bearing surface $A = 2\pi[r_1 - r_2] \times dr$

Load transmitted $(dW) = P \times A = P \times 2\pi r\, dr$

Frictional Resistance $(F_r) = \mu \times dW = \mu \times P \times 2\pi r\, dr$

Frictional Torque $(T_r) = F_r \times r = \mu \times P \times 2\pi r\, dr \times r$

integrating the above equation with Limits r_2 to r_1

$$T = \int_{r_2}^{r_1} 2 \times \pi \times \mu \times P \times r^2\, dr = 2\pi\mu P \times \left[\frac{r^3}{3}\right]_{r_2}^{r_1} = 2\pi\mu P\left[\frac{r_1^3 - r_2^3}{3}\right]$$

Now putting the value of P in the above equation we have

$$T = 2 \times \cancel{\pi} \times \mu \times \frac{W}{\cancel{\pi}(r_1^2 - r_2^2)} \times \frac{r_1^3 - r_2^3}{3} = \frac{2}{3} \times \mu \times W \times \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

in a multicollar bearing the $P = \frac{W}{n \times \pi \times [r_1^2 - r_2^2]}$ n = no. of collar

(ii) uniform wear →

for uniform wear $P \times r$ = constant $\Rightarrow P = \frac{C}{r}$

Load transmitted $(dW) = P \times A = \frac{C}{\cancel{r}} \times 2\pi\cancel{r}.\, dr$

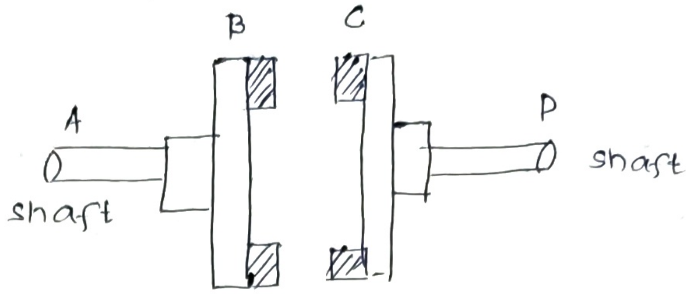
Total load transmitted $W = \int_{r_2}^{r_1} 2\pi C\, dr = 2\pi C[r_1 - r_2]$

Frictional Torque $(T_r) = F_r \times r = \mu \times dW \times r = \mu \times 2\pi C\, dr.\, r$

Total Torque $(T) = \int_{r_2}^{r_1} 2 \times \pi \times \mu \times C \times r.\, dr = \frac{1}{2} \times \mu \times W \times (r_1 + r_2)$

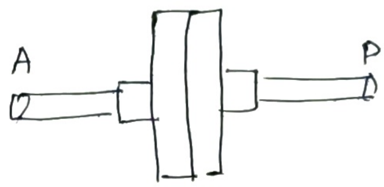
Clutches

It is a device that transmite Rotary motion of one shaft to another.



(Before engagement)

When shaft A Rotates at some speed Disc B which is mounted on the shaft A Rotates at the same speed But the shaft D is stationary and disc mounted on it also Remains stationary.



(After engagement)

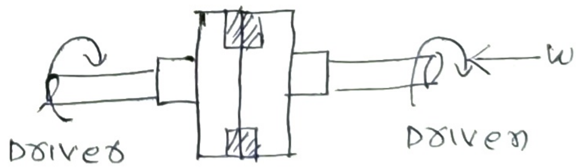
The axail Force W acts on D press the disc C to B and due to friction lining B transmite the Rotary motion to C and D also Rotates. The speed of driven shaft D depends upon the magnitude of axail Force applied. Motion transmitted by Friction so it is called Frictional Clutches.

Properties of Friction lining Material

(1) high value of co-effcient of Friction
(2) high thermal Resistance
(3) very high wear Resistance
(4) should not be affected by moisture.

The Frictional clutches are very important and divided into

(1) Disc / Plate clutch



W = axail load

μ = co-effcient of Friction

Tangential Force (F) = $\mu \times W$

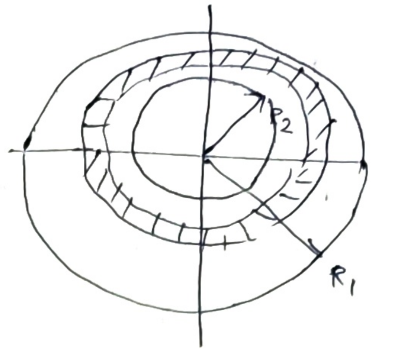
Frictional Torque (T) = $\mu \times W \times r$

Power lost due to Friction (P) = $T \times \omega = T \times \frac{2 \times \pi \times N}{60}$

1) uniform pressure →

Let's consider two Frictional surface maintained in contact by an axail load W.

r_1 and r_2 external and internal Radius of the Frictional surface consider an elementary ring of radius r and thickness dr



Area of contact $(dA) = 2\pi r\, dr$

axail force $(dw) = P \times 2\pi r\, dr$

Frictional Force $= \mu \times dw = \mu \times p \times 2\pi r\, dr$

Frictional Torque $= F_r \times r = \mu \times p \times 2\pi r\, dr \times r$

As we know in case of uniform pressure $p = \dfrac{W}{\pi[r_1^2 - r_2^2]}$

Frictional Torque $= \mu \times p \times 2\pi r^2 dr$

Total torque $= \int_{r_2}^{r_1} \mu \times p \times 2\pi \times r^2 dr = 2 \times \pi \times \mu \times p \times \int_{r_2}^{r_1} r^2 dr$

$= 2 \times \pi \times \mu \times p \times \left[\dfrac{r^3}{3}\right]_{r_2}^{r_1} = 2 \times \pi \times \mu \times p \times \left[\dfrac{r_1^3 - r_2^3}{3}\right]$

$= 2 \times \pi \times \mu \times \dfrac{W}{\pi[r_1^2 - r_2^2]} \times \left[\dfrac{r_1^3 - r_2^3}{3}\right] = \dfrac{2}{3} \times \mu \times W \times \left[\dfrac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right] \quad \ldots (I)$

As previously we know $T = n \times \mu \times W \times R \quad \ldots (II)$

compareing equation (I) and (II) $R = \dfrac{2}{3}\left[\dfrac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$ (mean Radius)

2) uniform wear Theory →

in case of uniform wear theory $P \times r =$ constant.

$P = \dfrac{C}{r}$

axail load $(dw) = \dfrac{C}{r} \times 2\pi r\, dr = 2\pi C\, dr$

Total load $W = \int_{r_2}^{r_1} 2\pi C\, dr = 2\pi C[r_1 - r_2] \Rightarrow C = \dfrac{W}{2\pi[r_1 - r_2]}$

Frictional Force $= \mu \times dw = \mu \times 2\pi C\, dr$

Frictional Torque $= F_F \times r = \mu \times 2\pi C\, dr \times r$

Total torque $T = \int_{r_2}^{r_1} \mu \times 2\pi C \times r\, dr = 2\pi \times \mu \times C \times \left[\dfrac{r^2}{2}\right]_{r_2}^{r_1}$

$= 2 \times \pi \times \mu \times C \times \left[\dfrac{r_1^2 - r_2^2}{2}\right]$

$= \pi \times \mu \times C \times [r_1^2 - r_2^2]$

$= \pi \times \mu \times \dfrac{W}{2\pi[r_1 - r_2]} \times [r_1 + r_2][r_1 - r_2] = \dfrac{1}{2} \times \mu \times W \times (r_1 + r_2)$

As we know total torque equation $T = n \times \mu \times W \times R$

comparing both equation $R = \frac{r_1 + r_2}{2}$ (mean Radius)

n = no. of Pairs of Frictional surface.

Note:

(*) For a single disc or Plate clutch normally both sides are effective so $n = 2$

(*) Intensity of pressure is maximum at inner and minimum at outer

$P_{max} \times r_2 = C$ and $P_{min} \times r_1 = C$

(*) The Average pressure $= \frac{W}{\pi [r_1^2 - r_2^2]}$

(*) The uniform pressure theory gives higher Frictional Torque than uniform wear Theory.

Multiple Disc clutch

It is generally used when large torque is to be transmitted.

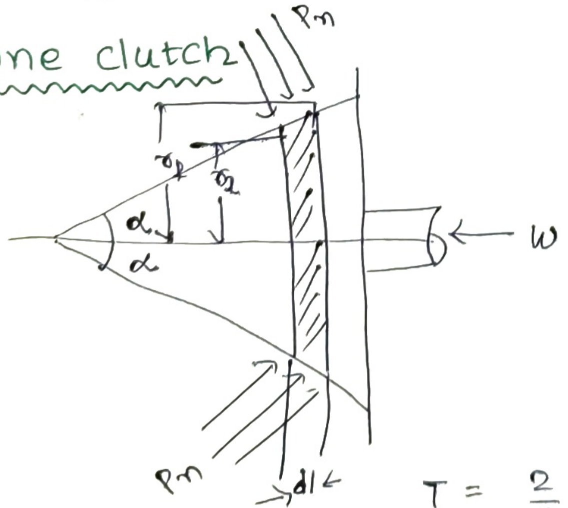
n_1 = no. of discs on driving shaft

n_2 = no. of discs on driven shaft

No. of pair of contact $= n_1 + n_2 - 1$

Total Torque $(T) = n \times \mu \times W \times R$

Cone clutch



This Analysis is similar to the Analysis of a Trapezodial Pivot bearing

P_n = intensity of pressure

r_1 and r_2 = outer and inner Radius of Friction surface

$$T = \frac{2}{3} \times \mu \times W \times \operatorname{cosec}\alpha \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right] \quad \text{(uniform pressure)}$$

$$T = \frac{1}{2} \times \mu \times W \times \operatorname{cosec}\alpha \left[r_1 + r_2\right] \quad \text{(uniform wear)}$$

Questions

(1) In a multiple disc clutch having 10 pairs of contact in the driving shaft and 15 pair of contact in the driven shaft. Total pair of contact

(a) 25 ✓(b) 24 (c) 15 (d) 10

(2) The Frictional torque transmitted by a disc / plate clutch is same that of

(a) Flat Pivot ✓(b) Flat collar (c) conical pivot (d) trapezoidal pivot

(3) The Frictional Torque transmitted by a cone clutch is same that of

(a) Flat pivot (b) flat collar (c) conical pivot ✓(d) trapezodial pivot

Problems

(Q) A shaft has a number of collar integral to it. The external diameter of the collar is 400 mm and shaft diameter is 250 mm. If the intensity of pressure is 0.35 N/mm^2 (uniform) and co-effient of Friction is 0.05 Find (1) Power absorbed at 105 r.p.m carrying a load of 150 kN (2) no. of collar Required

Ans → $r_1 = 400mm$, $r_2 = 250mm$, $P_1 = 0.35 N/mm^2$, $\mu = 0.05$, $N = 105$, $W = 150kN$

Using uniform pressure $(T) = \frac{2}{3} \times \mu \times W \times \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$

$= \frac{2}{3} \times 0.05 \times 150 \times 10^3 \times \left[\frac{400^3 - 250^3}{400^2 - 250^2}\right] = 1240 N\text{-}m$

Power absorbed $(P) = T \times \omega = T \times \frac{2 \times \pi \times N}{60}$

$= 1240 \times \frac{2 \times 3.14 \times 105}{60} = 13.64 kW$

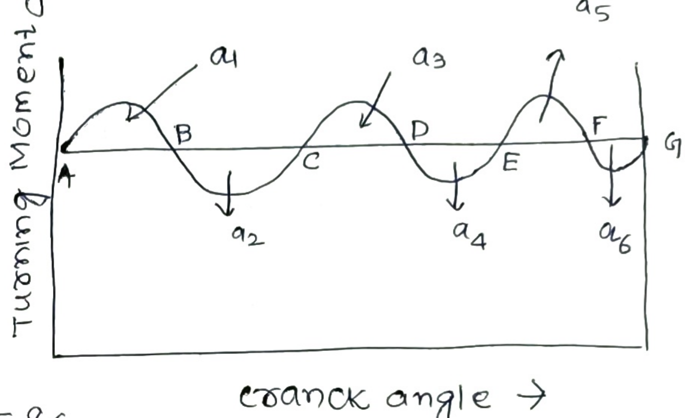
As we know $P = \frac{W}{n \times \pi [r_1^2 - r_2^2]}$ ⇒ $n = \frac{W}{P[\pi [r_1^2 - r_2^2]]}$

$= \frac{150 \times 10^3}{0.35 \times 3.14 \times [400^3 - 250^3]} = 5.6 \approx 6$

—×—×—

Fluctuation of Energy

The Fluctuation of energy may be determined by the turning moment diagram from one complete cycle of operation. The difference between the maximum and minimum energy is known as Fluctuation of energy (Maximum).
The variation of Energy above or below the Mean torque line is called Fluctuation of energy.



let the Energy at A = E

Energy at B = $E + a_1$

Energy at C = $E + a_1 - a_2$

Energy at D = $E + a_1 - a_2 + a_3$

Energy at E = $E + a_1 - a_2 + a_3 - a_4$

Energy at F = $E + a_1 - a_2 + a_3 - a_4 + a_5$

Energy at G = $E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$

Let's say the maximum energy at B = $E + a_1$

Let's say the minimum energy at E = $E + a_1 - a_2 + a_3 - a_4 + a_5$

Maximum Fluctuation of Energy = $\text{Maximum}_E - \text{minimum}_E$

Co-efficient of fluctuation of Energy

$$C_E = \frac{\text{Maximum Fluctuation of energy}}{\text{work done per cycle}}$$

Work done = $T_{mean} \times \theta$, $T_{mean} = \dfrac{60 \times P}{2\pi N}$,

Flywheel

A flywheel used in machines servers as a Reservior which stores energy during the period when the supply is more than it's requirment and release it during the period when the requirment of energy is more than of supply.

taking the example of a four stroke engine as we know power is generated during the expansion stroke which is more than the requirment and Release it during the period of compression, exhaust and inlet stroke. so during this period it supplies energy to the crankshaft for uniform Rotation of cran-
we observe that when flywheel absorb energy the speed of the engine increases and when it release energy speed decreases. Hence a flywheel does not maintain constant speed but it

controls the speed variation caused by the fluctuation of energy turning moment during one cycle of operation.

Co-efficient of fluctuation of speed

The difference between the maximum and minimum speed during a cycle is called maximum fluctuation of speed. When it is divided to the mean speed is called co-efficient of fluctuation of speed.

N_1 and N_2 be the maximum and minimum speed in r.p.m

Maximum fluctuation $= N_1 - N_2$

Mean speed $= \frac{N_1 + N_2}{2}$

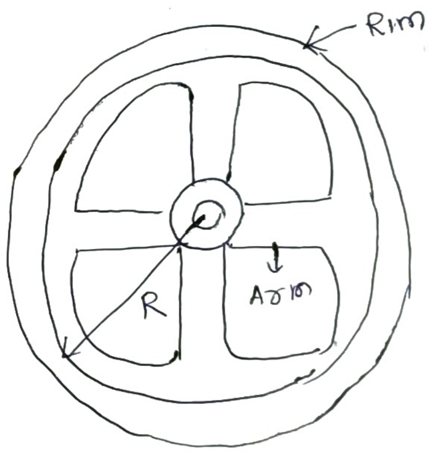
So co-efficient of fluctuation of speed

$$C_S = \frac{N_1 - N_2}{\frac{N_1 + N_2}{2}} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

In terms of angular velocity

$$C_S = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2}$$

Energy stored in a Flywheel



m = mass of flywheel

K = Radius of gyration

I = mass moment of inertia $= m \times K^2$

N_1 and N_2 are the maximum and minimum speed in r.p.m

ω_1 and ω_2 " " " " " Angular speed

$$C_S = \frac{N_1 - N_2}{N} = \frac{\omega_1 - \omega_2}{\omega}$$

Kinetic energy $= \frac{1}{2} \times I \times \omega^2$

$= \frac{1}{2} \times mK^2 \times \omega^2$

As we know the speed of flywheel changes from ω_1 to ω_2

$$\Delta E = \frac{1}{2} \times I \times \omega_1^2 - \frac{1}{2} \times I \times \omega_2^2 = \frac{1}{2} \times I \times (\omega_1^2 - \omega_2^2) = \frac{1}{2} \times I \times (\omega_1 + \omega_2)(\omega_1 - \omega_2)$$

$$= \frac{1}{2} \times I \times (\omega_1 + \omega_2) \times (\omega_1 - \omega_2) = I \times (\omega_1 - \omega_2) \times \frac{(\omega_1 + \omega_2)}{2} = I \times (\omega_1 - \omega_2) \times \omega$$

$$\Delta E = I \times \omega^2 \times \frac{\omega_1 - \omega_2}{\omega} = \boxed{I \times \omega^2 \times C_S} = mK^2 \times \omega^2 \times C_S = 2 \times K.E \times C_S$$

$$= \boxed{2 E C_S}$$

Governors

The main function of governor is to regulate the mean speed of the engine when there is variation of load. when the load on the engine increases it's speed decreases therefore it becomes neccary to increse the supply of working fluid on the other hand when the load on the engine decreases it's speed increases and thus less working fluid is required. The governor automatically controls the supply of working fluid to the engine under varying load conditions. It keeps the mean speed with certain limits.

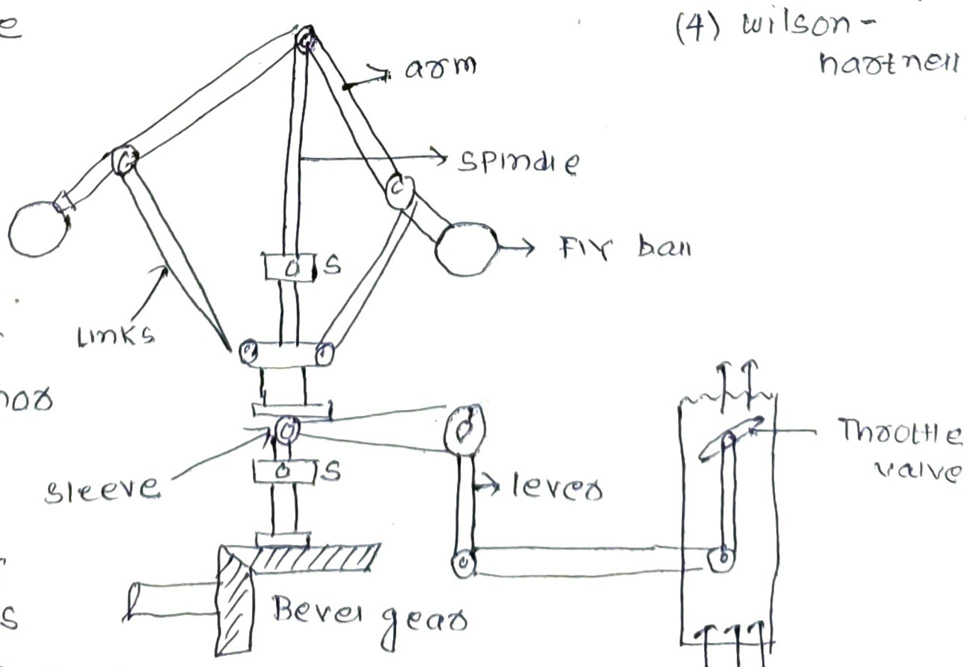
Types of Governors

These are classified as (1) centrifugal governors
(2) inertia governors

- centrifugal governors
  - Pendulum type
    - watt governor
  - Loaded type
    - Dead weight
      - Porter governor
      - Proell governor
    - spring controlled
      - (1) Hartnell
      - (2) Hartung
      - (3) Pickering
      - (4) wilson-hartnell

Centrifugal governor

It is based upon the principle of balancing the centrifugal force on the Rotating balls by an equal and oppsite Radial Force known as controlling Force. It consists of two balls of equal mass called governor balls or Fly balls. These balls Revolve around a spindle which is driven by the Bevel gear. The arm is



Pivoted to the spindle so that the ball may rise or Fall down as the Revolve around the vertical axis.

when the load on the engine increases the engine and governor speed decreases This Result in decresing centrifugal Force Hence the ball move inward and sleeve moves downward This downward movement of the sleeve operates the throotle valve so that it opens and supply the working fluid thus speed of the engine increases.

When the load on the engine decreases the engine and governor speed increases which Result in increase in the centrifugal force Thus the balls moves outward and the sleeve moves upward which reduce the supply of working fluid and hence speed decreases.

Terms used in governor

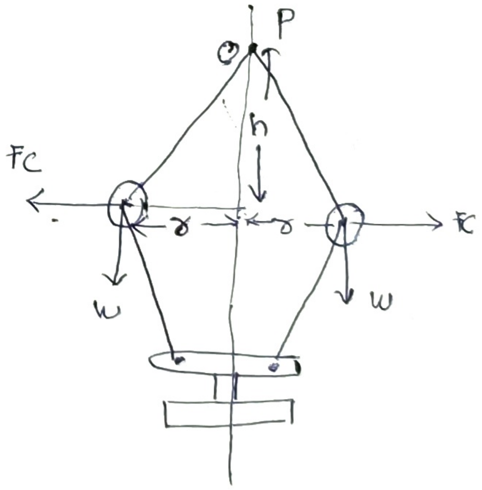
(1) height of governor → vertical distance from centre of the ball to a point where the axes of the arm intersect.

(2) Equilibrium speed → It is the speed at which the governor balls, arms, etc are in complete equilibrium.

(3) Sleeve lift → vertical distance which the sleeve travel due to change in equilibrium speed.

(4) Maximum and minimum equilibrium speed → The speed at the maxim and minimum radius of rotation of the balls without tends to move either way.

(1) Watt Governor



Simplest Form of a cetrifugal governor is watt governor.
considering one part of governor

taking moment at O

$$F_c \times h = w \times r$$

F_c = centrifugal $= m\omega^2 r$

$$m\omega^2 r \times h = m \times g \times r$$

$$\Rightarrow h = \frac{g}{\omega^2}$$

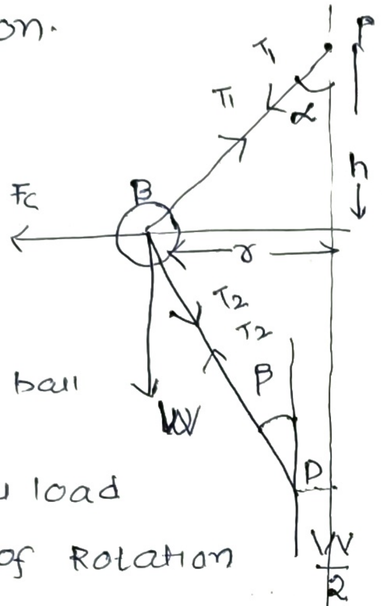
If we use $\omega = \frac{2\pi N}{60}$

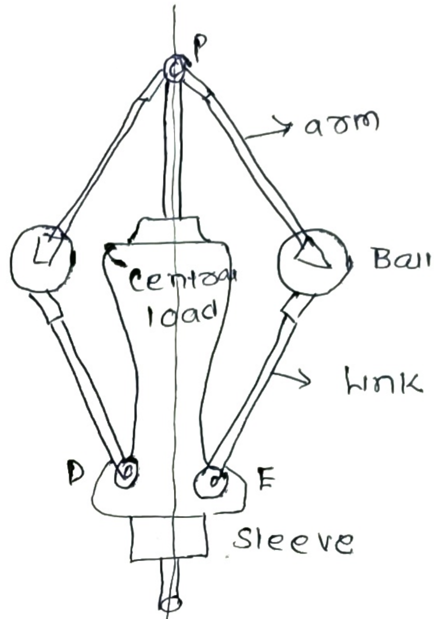
$$\Rightarrow h = \frac{(60)^2 g}{4 \times \pi^2 \times N^2} = \frac{895}{N^2} \text{ meter}$$

so $\boxed{h \propto \frac{1}{N^2}}$

Porter Governor

This is the modification of watt governor with central load attached to the sleeve. The load moves up and down with the central load. This additional Force increases the speed of Revolution.





$F_c = m \times \omega^2 \times r$

m = mass of ball

M = mass of central load

r = radius of Rotation

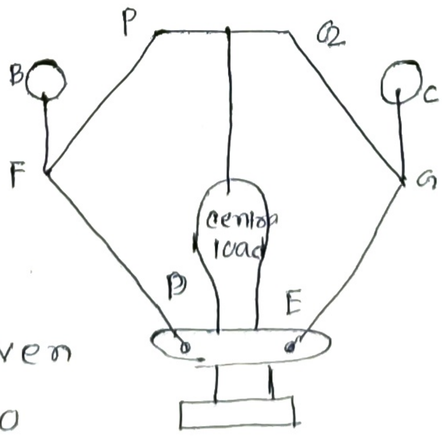
using the method of Resolution we get $N^2 = \frac{m+M}{m} \times \frac{895}{h}$

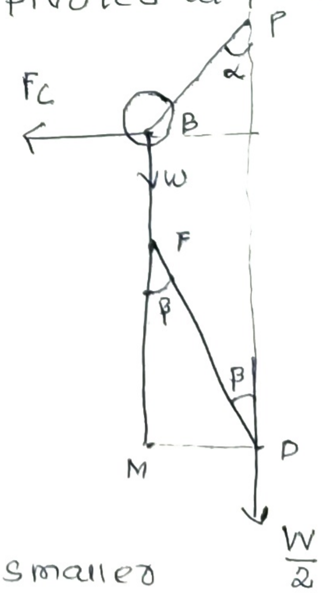
on comparing the equation with watt governor we find that the mass of central load M increases the height of the governor in the Ratio of $\frac{m+M}{m}$

Proell Governor

The proell governor has the ball Fixed at B and C to the extension of the link DF and EG. The arms are pivoted at P & Q.

$$N^2 = \frac{FM}{BM}\left[\frac{m+M}{m}\right] \times \frac{895}{h}$$





From this equation we observe that equilibrium speed decreses for a given value of m, M and h. so in order to have same equilibrium speed balls of smaller masses are used in proell governor.

Sensitiveness of Governor

consider two governor A and B running at same speed when the speed increases or decreases by some amount the lift of the sleeve A is greater than B. Then governor A is more sensitive than B.

It is the Ratio of difference between the maximum and minimum equilibrium speed to the Mean speed.

$$\text{Sensitiveness} = \frac{N_2 - N_1}{N} = \frac{N_2 - N_1}{\frac{N_1 + N_2}{2}} = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

Stability of Governor

A governor is said to be stable when for every speed within the working range there is only one radius of rotation of the governor ball. If equilibrium speed increases the radius of governor ball also increases. otherwise it is called unstable.

Isochronous Governors

If the equilibrium speed is Remains constant for all radii of rotation of the governor ball. neglecting friction. A porter governor can't be ischronous.

Hunting

A governor is said to be hunt if the speed of the engine fluctuates continously above and below the mean speed. This is due to the too sensitive governor which changes the fuel supply by a large amount.

Effort and Power of a Governor

The effort of a governor is the mean force exerted at the sleeve for a given percentage of change in speed

The power of a governor is the work done at the sleeve for a given percentage of change in speed.

Power = Mean effort × lift of sleeve.

×××

Questions

(1) In turning moment diagram, the variation of energy above and below the mean resisting torque line is called

(a) Fluctuation of energy (b) co-efficient of Fluctuation of energy

(c) maximum Fluctuation of energy (d) All

(2) The maximum Fluctuation of Energy of a flywheel is equal to

(a) $I \times \omega^2 \times C_s$ (b) $2 \times E \times C_s$ (c) $I \times \omega \times (\omega_1 - \omega_2)$ (d) All

(3) A hartnell governor is a ______ governor

(a) spring loaded (b) Dead weight (c) inertia (d) pendulum type

(4) ______ governor is called gravity operated governor.

(a) watt (b) proell (c) porter (d) hartnell

(5) A governor is said to be hunt if the speed of the engine

(a) Remain constant (b) is above mean speed (c) below mean spee

(d) continously Fluctuates above and below the mean speed

(6) when the sleeve of a Porter governor moves upward the governor speed

(a) increases (b) decreases (c) constant (d) All

(7) The Ratio of height of a porter governor to watt governor

(a) $\frac{m}{m+M}$ (b) $\frac{M}{m+M}$ (c) $\frac{m+M}{m}$ (d) $\frac{m+M}{M}$

(8) A hunting governor is

(a) more stable (b) less stable (c) too sensitive (d) less sensitiv

Solved problems

(Q) The mass of flywheel of a engine is 65 kg and the radius of gyration is 1.8 m. The Fluctuation of energy is 56 kN-m. If the mean speed of the engine is 120 r.p.m. Find the Maximum and minimum speed ?

Ans → $m = 65$ kg $\quad \Delta E = 56 \times 10^3$ N-m

$K = 1.8$ m $\quad N = 120$ r.p.m

As we know $I = m \times k^2 = 65 \times (1.8)^2 = 210.6$

and $AE = I \times \omega^2 \times C_s$

$\Rightarrow \quad 56 \times 10^3 = 210.6 \times \left(\frac{2\pi N}{60}\right)^2 \times \left(\frac{N_1 - N_2}{N}\right)$

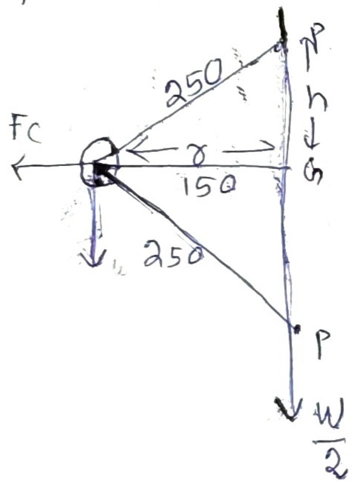
$\Rightarrow \quad 56 \times 10^3 = 210.6 \times \left(\frac{2 \times 3.14 \times 120}{60}\right)^2 \times \left(\frac{N_1 - N_2}{120}\right)$

$\Rightarrow \quad N_1 - N_2 = 2 \quad \ldots (1)$

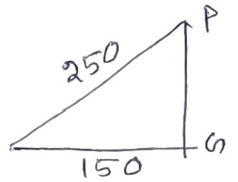
As we know $\frac{N_1 + N_2}{2} = 120 \Rightarrow N_1 + N_2 = 240 \quad \ldots (11)$

From equation (1) and (2) $N_1 = 121$, $N_2 = 119$ r.p.m

(2) a Porter governor has equal arms each 250 mm long and pivoted on the axis of rotation. each ball has mass of 5 kg and mass of central load is 25 kg. The radius of rotation of the ball is 150 mm Find the speed of governor ?



From the Right angle triangle



$h^2 = p^2 + b^2$

$250^2 = PG^2 + 150^2$

$PG^2 = 250^2 - 150^2$

$\Rightarrow PG = 0.2\,m$

Now $N^2 = \frac{m + M}{m} \times \frac{895}{h}$

$N^2 = \frac{5 + 25}{5} \times \frac{895}{0.2} = 17,900$

$\Rightarrow N = \sqrt{17,900} = 133.8$ r.p.m

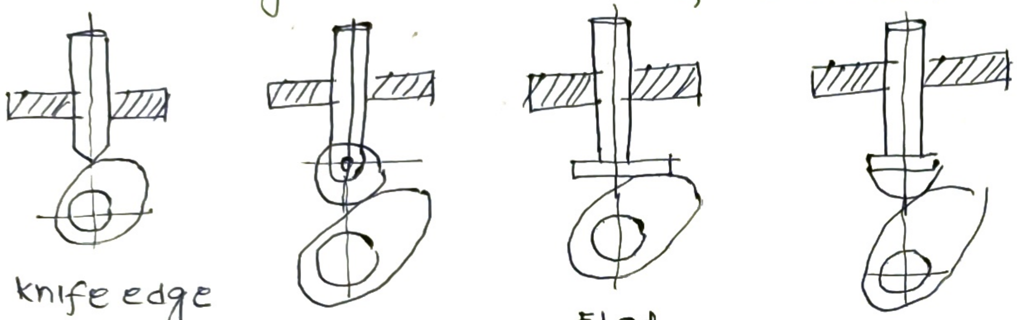
—x—x—x

Cams

Cam is a rotateing machine element which gives recipocating or oscillating motion to another element known as Follower There is line contact between them so they form higher pair. cam generally rotate at a uniform speed and the Follower motion is to be predetermined according to the shape of the cam.

Classification of Followers

1) According to the surface of contact →



In case of knifeedge Follower the contacting end has a sharp knife and sliding motion takes place at the contact surface it is seldom used due to less contact betn members.

In case of Roller Follower contacting end is a roller which is pure rolling motion takes place here the rate of wear is gradually reduceed. It is used in air-craft, oil engine etc.

In case of Flat faced follower the contacting end is Flat Faced due to which The side thrust is much reduced. The Relative motion between them is sliding in nature it is widely used in case of Automobile engines.

In case of spherical Follower the contacting end is a sphere and as flat faced Follower produces high stress The spherical faced Follower is widely used.

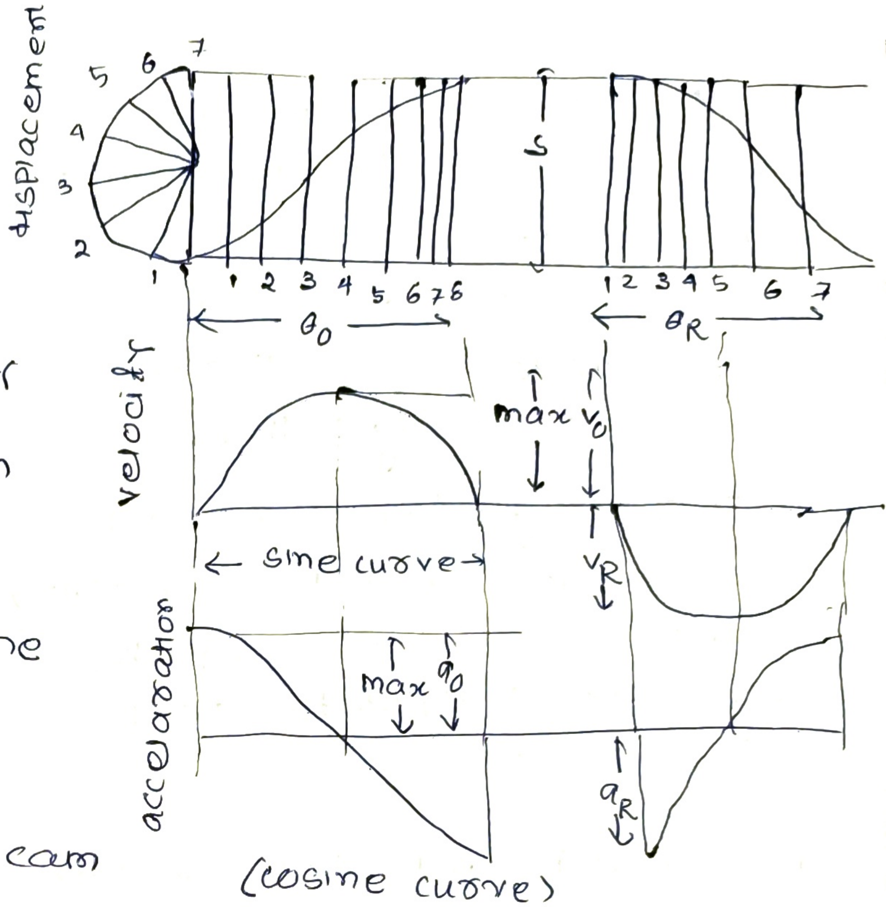
2) According to the motion of the Follower →

The motion is either Recipocating or oscillating and during these conditions the cam Rotates at a uniform speed.

3) Path of motion of Follower →

When the motion of the Follower is such that It is

. The displacement curve is drawn by using a semi-circle and divide it into equal no. of section. Since During the simple harmonic motion the velocity diagram is a sine curve and the acceleration diagram is a cosine curve and S is the stroke and lift of the Follower and θ_0 and θ_R be the Outward stroke and Return stroke.



w = angular velocity of the cam

$t_0 = \frac{\theta_0}{w}$ and $t_R = \frac{\theta_R}{w}$

$v_0 = \frac{\pi \times w \times S}{2 \times (\theta_0)}$ $\quad V_R = \frac{\pi \times w \times S}{2 \times (\theta_R)}$

$a_0 = \frac{\pi^2 \times w^2 \times S}{2 \times (\theta_0)^2}$ $\quad a_R = \frac{\pi^2 \times w^2 \times S}{2 \times (\theta_R)^2}$

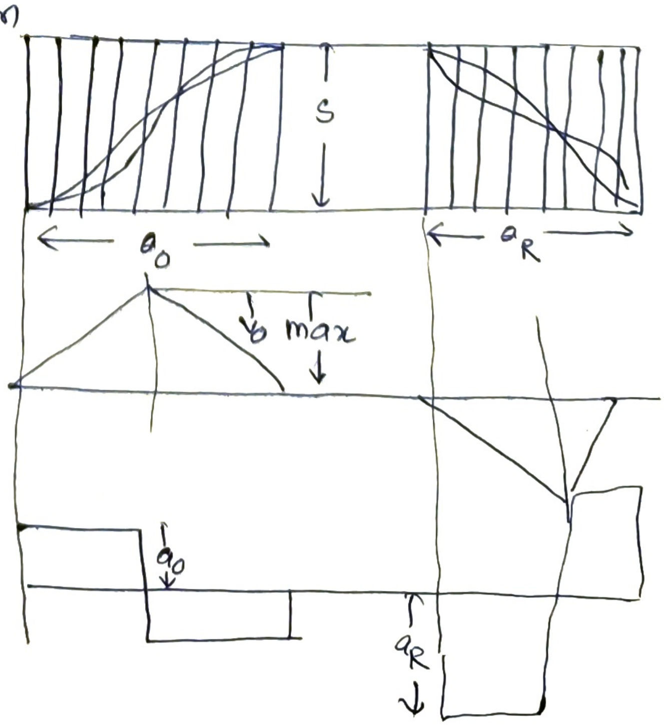
(3) When Follower motion uniform acceleration or Retardation →

here the displacement diagram is Parabolic curve and again

$t_0 = \frac{\theta_0}{w}$ and $t_R = \frac{\theta_R}{w}$

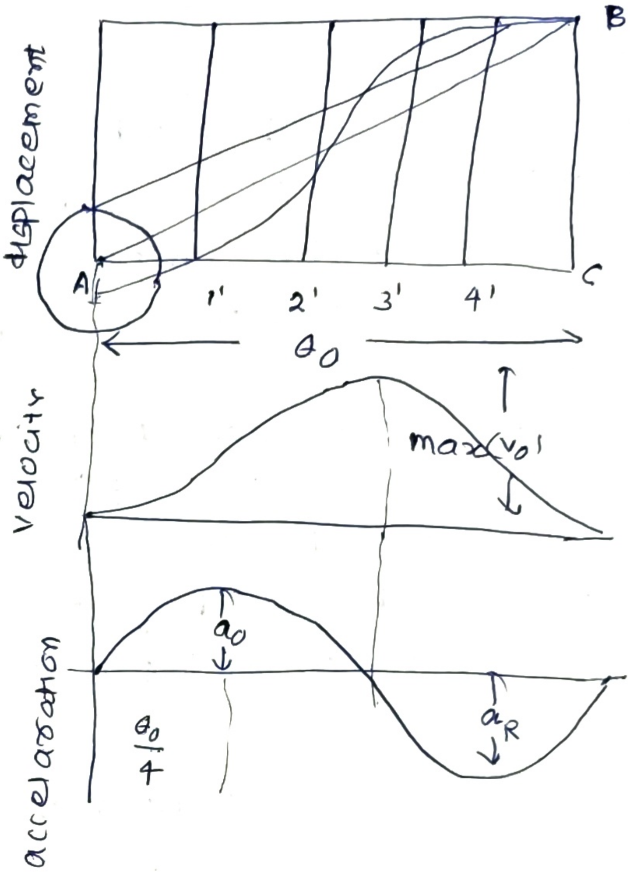
$v_0 = \frac{2 \times w \times S}{\theta_0}$ and $v_R = \frac{2 \times w \times S}{\theta_R}$

$a_0 = \frac{4 \times w^2 \times S}{(\theta_0)^2}$ and $a_R = \frac{4 \times w^2 \times S}{(\theta_R)^2}$



here uniform Retardation during return stroke and uniform acceleration during outward stroke.

(4) Cycloidal Motion →



It is the diagram which represent the cycloidal motion we know cycloid is the curve traced by a point on a circle when the circle rolls without slipping on a straight line.

The straight line is a stroke of the Follower which is translating and the circumference of the Rolling circle is equal to stroke of the Follower.

It is used for high speed motion of the follower.

$$v_0 = \frac{2 \times w \times s}{\theta_0} \qquad v_R = \frac{2 \times w \times s}{\theta_R}$$

$$a_0 = \frac{2 \times \pi \times w^2 \times s}{(\theta_0)^2} \qquad a_R = \frac{2 \times \pi \times w^2 \times s}{(\theta_R)^2}$$

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