# LECTURE NOTES

# ON
# STRENGTH OF MATERIAL

## 3$^{RD}$ SEMESTER

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# Strength of Material

Chapter's include →

(1) Simple stress and strain
(2) Principal stresses
(3) Shear Force and Bending Moment
(4) Pure Bending
(5) Deflection of Beams
(6) Torsion of circular shaft
(7) closed coiled helical spring
(8) Theory of column
(9) Strain Energy
(10) shear stress distribution Diagram

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# Strength of Material

Important terms used →

(1) Elasticity → It is the property by virtue of which a material deformed under the load is enabled to return to its original dimension when the load is Removed.
ex → Steel, Aluminium, copper

(2) Plasticity → It is the property by virtue of which a material is deformed under the load is unable to return to its original state after the Removal of load.

(3) Ductility → It is the property which permits a material to be drawn out longitudinally to a Reduced section under the action of tensile force. example → copper, Aluminium, steel

(4) Brittleness → It is the lack of ductility, we can called the material to be brittle it breaks with little elastic deformation and without significant plastic deformation. example → cast iron, ceramic, concrete

(5) homogenous Material → A material is said to be homogeneous Material which has uniform composition and uniform properties through out. Metals, alloy, ceramics

(6) Isotropic → when the properties of the materials are same in all direction the material is called isotropic.

(7) anisotropic → when the properties of the material vary with different crystallographic orientation.

## Stress

The internal Resistance offered by the body to meet with the load is called stress.

i) simple stress/ direct stress → because it develops under direct loading conditions.

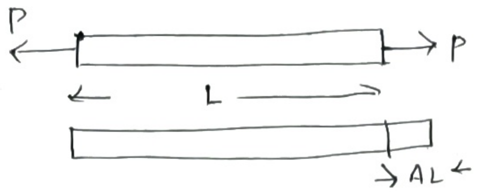
ii) Tensile stress, compressive stress, shear stress

iii) It is denoted by $(\sigma) = \frac{\text{Load}}{\text{Area}} = N/m^2$

## Strain

Strain is the deformation produced due to stress. It can be defined as change in length to original length. It is denoted as (e)

$$e = \frac{\Delta L}{L}$$




In case of shear strain it is measured by the angle through which the material distort. Let the angle is $\theta$

$$e_s = \tan\phi$$

## Volumetric strain

It is defined as the Ratio between change in volume to the original volume of the body.

$$e_v = \frac{\Delta V}{V}$$

## Hooke's Law

Robert hooke discovered experimentally that within the elastic limit stress varies directly as strain.

$$\sigma \propto e$$

$$\Rightarrow \sigma = Ee$$

$$\Rightarrow E = \frac{\sigma}{e} = \text{Modulus of elasticity}$$

So it is defined as the Ratio between tensile stress to tensile strain.

## Modulus of Rigidity

It is the Ratio between shear stress to shear strain. denoted by C/G/N.

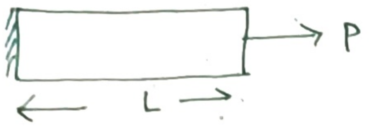
$$G = \frac{\tau}{\phi}$$

## Bulk/ volume modulus of elasticity

It is defined as the Ratio between normal stress to volumetric strain. denoted by K.

$$K = \frac{\sigma}{e_v}$$

## Elongation of a prismatic bar due to external load

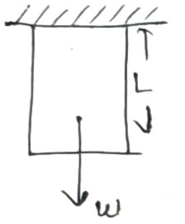



$$\text{elongation } (\Delta L) = \frac{P \times L}{A \times E}$$

P = load applied  L = Length

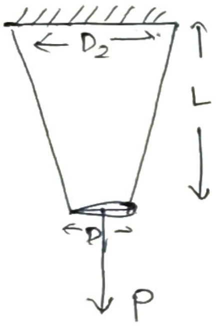
A = Area  E = Young's modulus of elasticity

## Elongation due to self weight ($\Delta$)




$$\Delta L = \frac{W \times L}{2 \times A \times E}$$

## Elongation / Deflection of tapered bar




$$\Delta = \frac{4 \times P \times L}{\pi E D_1 D_2}$$

## Poisson Ratio ($\mu$)

It is defined as the Ratio between lateral strain to longitudinal strain.

$\mu$ = 0 to 0.5 (uni-axial loading)

## Relationship between elastic constant

(1) $E = 2G(1+\mu)$

(2) $E = 3K(1-2\mu)$

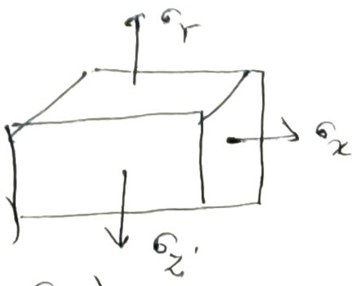
(3) $E = \dfrac{9KG}{3K+G}$

G = modulus of Rigidity

K = Bulk modulus of elasticity

E = Youngs modulus of elasticity

## Volumetric strain under tri-axial loading




$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\varepsilon_x = \frac{\sigma_x}{E} - \mu\left(\frac{\sigma_y}{E} + \frac{\sigma_z}{E}\right)$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \mu\left(\frac{\sigma_x}{E} + \frac{\sigma_z}{E}\right)$$

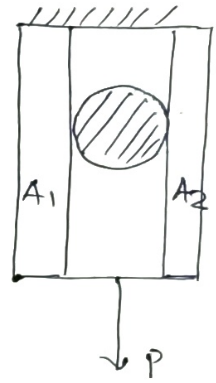
$$\varepsilon_z = \frac{\sigma_z}{E} - \mu\left(\frac{\sigma_x}{E} + \frac{\sigma_y}{E}\right)$$

## lateral vs longitudinal strain

$\frac{\Delta L}{L}$ = strain / primary strain / linear strain

when the load is applied the dimension of the body changes in all direction at Right angle to it's line of application the strain produced are called lateral / secondary / transverse strain. They are oppsite in nature.

## stresses in compound structure



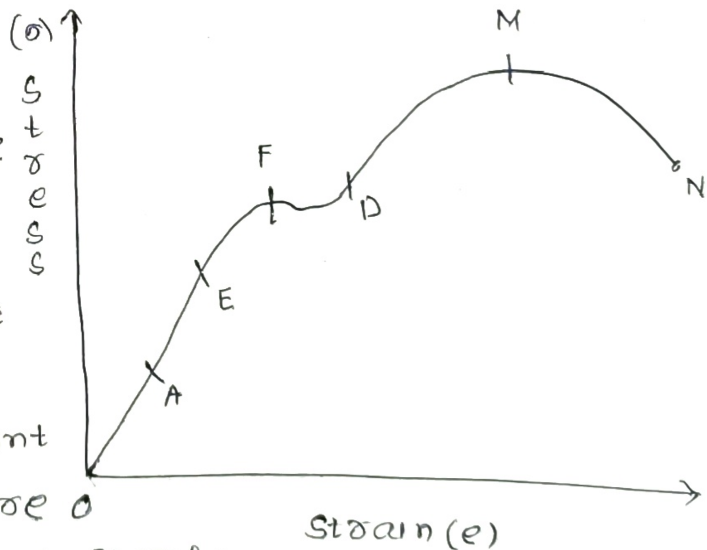

we have to assume that load is distibuted equally among the loads then

$P = \sigma_1 \times A_1 + \sigma_2 \times A_2$

so the strain produced are also equal

$e_1 = e_2 \Rightarrow \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$

## stress - strain diagram of ductile Material



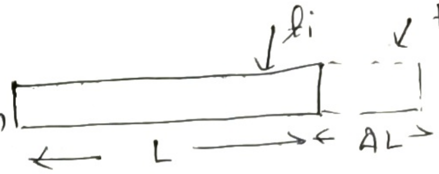

From the graph we observe that from the point (O to A) The curve is straight line means $\sigma \propto e$. which is obey's hook's law of elasticity. at the point E is called the elastic limit After the point E the graph shows a curve line and The point F is called upper yeild point where the Plastic deformation of Material starts and the point D is called lower yeild point where the material is totally converte to plastic. As going further we Reach at the point M which provide maximum strength of the Material called the point of ultimate stress. After the point M there is sharp decrese in the strength of Material and at point N the material breaks which is called necking or breaking point.

## stress - strain diagram for brittle Material

In case of brittle Material there is no appreciable change in stress and strain, no necking point etc.

## Thermal stress

If the temperature of a body is lowered or raised its dimension will decrese or increase corrospondingly. The stress developed under these condition called thermal stress.




$\alpha$ = co-effcient of linear expansion

$$\Delta L = \alpha L \Delta T = \alpha \times L \times (T_1 - T_f)$$

so strain produced $= \frac{\Delta L}{L} = \frac{\alpha \times \cancel{L} \times (\Delta T)}{\cancel{L}} = \alpha \Delta T$

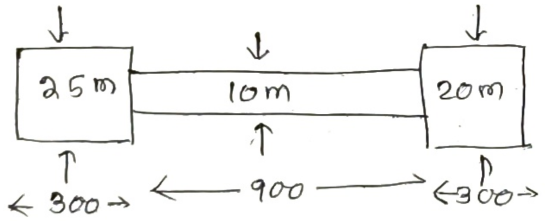
Thermal stress $= \alpha \Delta T E$

in case of uniform heating the thermal stress is 0.

### Problems

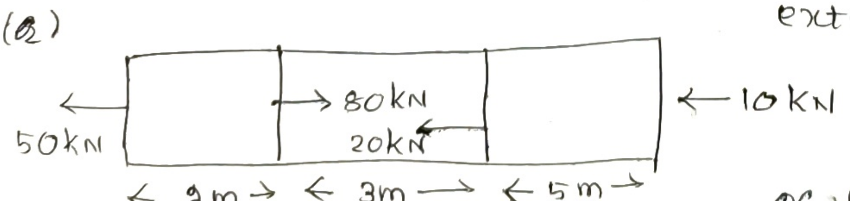
(Q) A square steel bar (20mm x 20mm) in section carries a axail load of 100 kN where $E = 2.14 \times 10^8$ kN/m$^2$ : Find stress, strain when the length of the bar is 30m

(Q)




Three bars having length and diameter as follows, If the bar is subjected to a tensile load of 15 kN. Find total extension ?

(Q)




Find the total elongation of the bar if all are having $E = 2 \times 10^5$ kN/m$^2$

(Q) Let's consider a bar subjected to the following condition

(1) diameter = 30 m (2) Load = 20N (3) Length = 5m

change in length = 0.5 m and change in diameter = 0.3 m

Find the value of Poisson's Ratio and all the elastic moduli

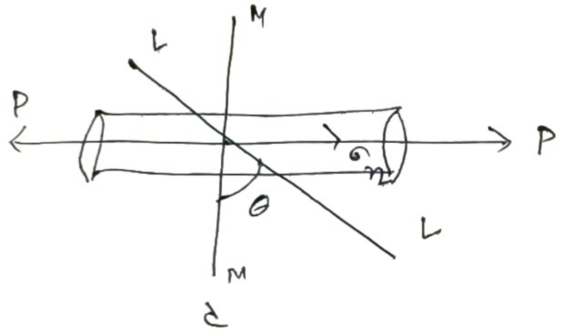
(Q) A steel rod having 15 m Long is at a temperature of 15°C Find the free exponsion of the length when temp is Raised to 65°C Find the temperature stress produced when

$\alpha = 12 \times 10^{-6}$ per °C $E = 200 \times 10^5$ kN/m$^2$

# Principal stresses and strains

There are always three mutually perpendicular plane along which the stresses are acting at a certain point on the body. These planes passes through a point in such a manner that the Resultant stress across them is totally a normal stress known as principal plane and the stress acting on the plane is called principal stress. The plane carrying the maximum normal stress is called major principal plane and the crossponding stress is called major Principal stress. Plane carrying minimum normal stress is called minor principal plane and stress is called minor principal stress.

## Case-1 (Stresses in a tensile member)



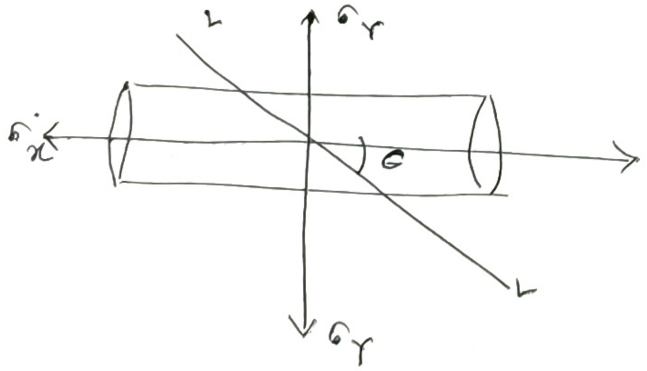

Normal stress $(\sigma_n) = \frac{P}{A}\cos^2\theta$

$= \sigma\cos^2\theta$

Shear stress/ tangential stress $(\tau) = \frac{\sigma}{2}\sin 2\theta$

This means that when a bar is subjected to tensile stress both Normal and shear stress occour

## Case-2 (Two mutually perpendicular direct stress)




Normal stress $(\sigma_n) =$

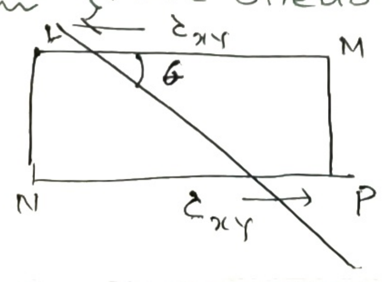
$$\frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

Shear stress $(\tau_s) =$

$$\frac{\sigma_x - \sigma_y}{2}\sin 2\theta$$

Resultant stress $\sigma_r = \sqrt{\sigma_n^2 + \tau_s^2}$

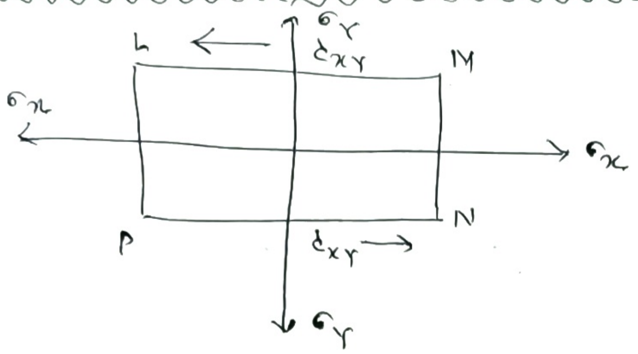
## Case-3 (Pure shear condition)




$\sigma_n = \tau_{xy}\sin 2\theta$ $\qquad \sigma_{n\,max} = \tau_{xy}$

$\tau = \tau_{xy}\cos 2\theta$ $\qquad \tau_{max} = \tau_{xy}$

# Two dimensional stress system




$$\sigma_n = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_t = \frac{\sigma_x-\sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

In order to findout the principal stress the maximum and minimum value of $\sigma_n$ must be taken

$$\frac{d}{d\theta}\sigma_n = 2\tau_{xy}\cos 2\theta - 2\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta$$

so we get $\tan 2\theta = \dfrac{2\times\tau_{xy}}{\sigma_x-\sigma_y}$

major principal stress $(\sigma_1) = \dfrac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2_{xy}}$

minor principal stress $(\sigma_2) = \dfrac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2_{xy}}$

$$\tau_{max} = \frac{\sigma_1-\sigma_2}{2}$$

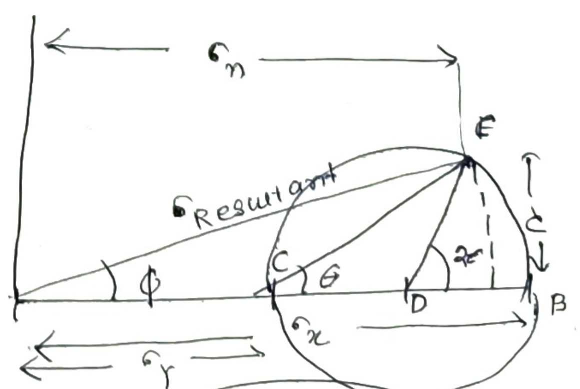
## Graphical Method of Finding Principal Stress

(1) Mohr's circle for like stresses →

take $\sigma_x$ and $\sigma_y$ be any scale as $\sigma_x > \sigma_y$

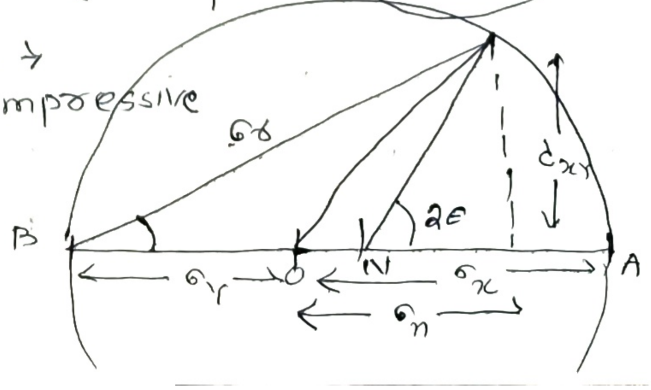
Draw $\sigma_x$ and $\sigma_y$

taking the obtaining Radius draw mohr's circle and taking angle $2\theta$ for the measurement bisects the circle then draw.




(2) Mohr's circle for unlike stresses →

Let $\sigma_x$ is tensile and $\sigma_y$ is compressive



using Formula we can also calculate the Radius of mohr's circle $R = \sqrt{\left(\frac{\sigma_x - \sigma_Y}{2}\right)^2 + \tau^2_{xY}}$

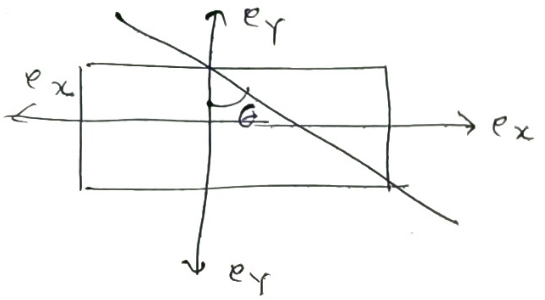
## Principal strain

It can be written as $e_1 = \frac{e_x + e_Y}{2} + \sqrt{(e_x - e_Y)^2 + \phi^2}$

$$e_2 = \frac{e_x + e_Y}{2} - \sqrt{(e_x - e_Y)^2 + \phi^2}$$

where $\phi$ is shear strain

If



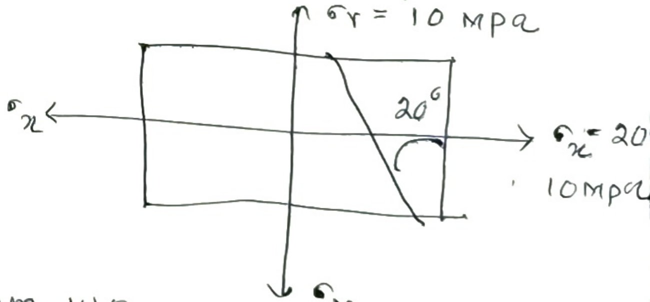

Total strain (e) = $e_x \cos^2\theta + e_Y \sin^2\theta$

(principal stresses problem)

## Problems

(Q) Find out the value of normal stress and shear stress and Resultant stress on a plane 20° with the major principal plane.

also solve it by mohr's circle method.




(Q) If for a significant stress system we have $\sigma_x = 25$ MPa $\sigma_Y = -15$ MPa

Find out the Radius of the mohr's circle?

(Q) In a strained material, normal stress on two mutually perpendicular planes are $\sigma_x$ and $\sigma_Y$ both alike accompanied by a shear stress $\tau_{xY}$. one of the principal stress will be zero if (a) $\tau_{xY} = \frac{\sigma_x \times \sigma_Y}{2}$ ~~(c)~~ $\tau_{xY} = \sqrt{\sigma_x \times \sigma_Y}$

(b) $\tau_{xY} = \sigma_x \times \sigma_Y$ (d) $\tau_{xY} = \sqrt{\sigma^2_x + \sigma^2_Y}$

# Shear Force and Bending Moment

a structure may be consisting of series of of beams linked together in some way to form a rigid structure.

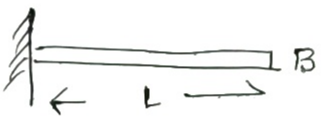
Beam → Beam is a structural member which is acted upon by a system of external load at Right angle to the axis.

Bending → Deformation produced in a bar by the load perpendicular to it's axis as well as Force couple acting in a plane passing through the axes.

If the plane passes through C.G of the section → Plane bending otherwise it is called oblique bending.

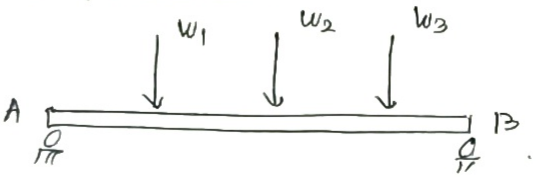
## Classification of Beams

1) Cantilever →



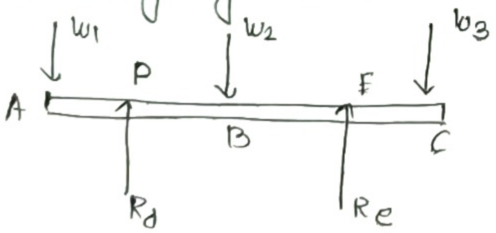

here one end is Fixed and other is Free

2) Simply supported →




whose ends are Freely rest on walls/ supports.

3) overhanging beam →

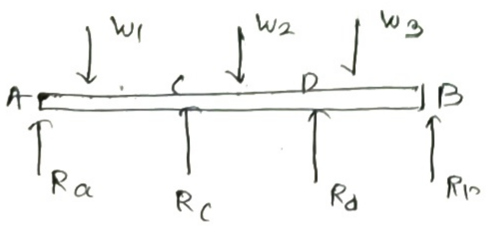



in this beam the supports are not situated at the ends.

4) Fixed beam →

whose Both the ends are Rigidly Fixed or built into supporting wall or coloumn.

(5) Continous beam →




A continous beam is one which has more than two supports. The inner supports are called intermediate support.

## Statically Determinate vs Statically Indeterminate

Those beams where using the equation of static equilibrium we can Analyze is called statically determinate Beam others beams are called statically indeterminate Beam.

Cantilever, simply supported, overhanging → Determinate beams

Fixed beam, continuous → In-determinate beams

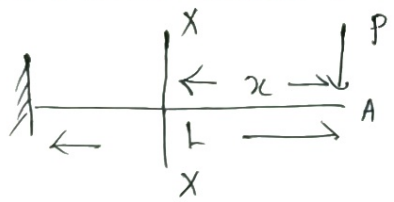
## Calculation of Shear Force and Bending Moment

Shear Force → a shearing force having an upward direction to the Right hand side of a section or downward to the left of the section is taken to be positive. Similarly other cases are negative.

Bending Moment → we consider clockwise bending moment to be positive and anti-clockwise bending moment to negative.

### Calculation

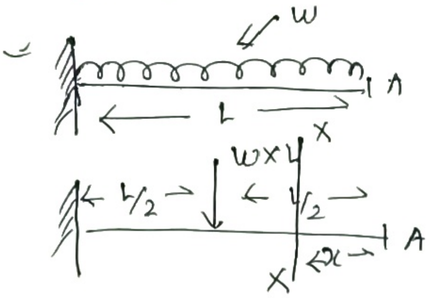
(1) Point load →




S.F at X-X = - P

B.M at X-X = - P × x

(2) U.D.L →

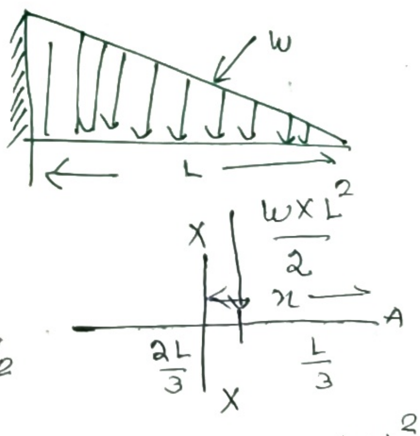



S.F at X-X = - w × L

B.M at X-X = - w × L × L/2

$= -\frac{wL^2}{2}$

(3) U.V.L →




S.F at X-X = $\frac{wx^2}{2}$

B.M at X-X = $\frac{wx^2}{2} \times \frac{x}{3} = \frac{-wx^3}{6}$

## Diagram consideration for Shear Force & Bending Moment

(1) Point load → S.F Diagram will consists of Rectangle

B.M Diagram will consists of inclined lines

(2) U.D.L → S.F Diagram will consists of inclined lines

B.M diagram consists of parabolic lines for U.D.L

(3) U.V.L → S.F diagram will consists of parabolic lines where as B.M diagram will consists of cubic / 3rd degree polynomial.

## Point of contraflexure

In the Bending moment diagram the point at which Bending moment changes sign and at the paticular point the value of Bending moment is zero. It is also known as point of inflexion.

## Relation between S.F and B.M

Using the load between shear force and Bending moment we can get the Relation $S.F = \frac{dM}{dx}$

M = Bending moment

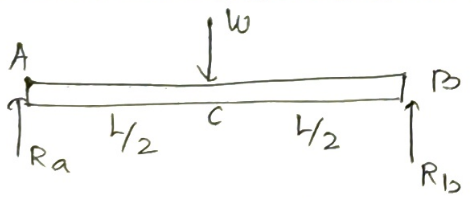
From the Relation we observe that

B.M will be maximum when $\frac{dM}{dx} = 0$ so $S.F = 0$

So we find that the point at which S.F is zero at the same point the B.M is either maximum or minimum.

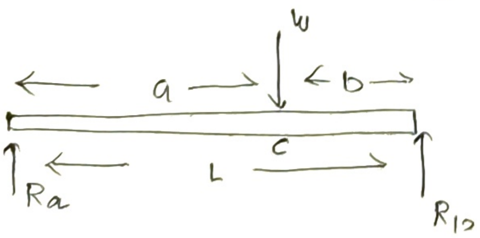
## Condition of maximum value

(1)



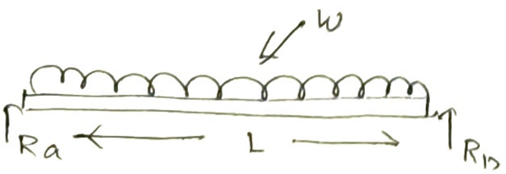

$S.F_{max} = \frac{w}{2}$ $B.M_{max} = \frac{w \times L}{4}$

(2)



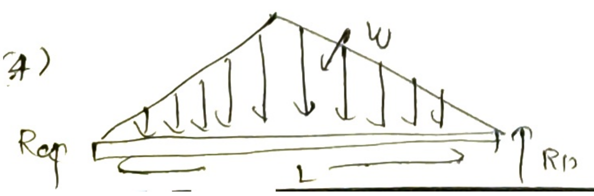

$S.F_{max} = \frac{w \times a}{L}$ $B.M_{max} = \frac{w \times a \times b}{L}$

(3)




$S.F_{max} = \frac{w \times L}{2}$ $B.M_{max} = \frac{w \times L^2}{8}$

4)




$S.F_{max} = \frac{w \times L}{4}$ $B.M_{max} = \frac{wL^2}{12}$

(Problems).

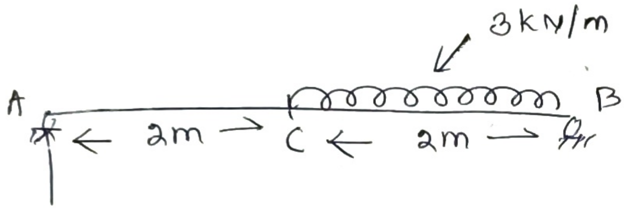
(1) If the Bending moment is given by the equation
$M(x) = x^2 + 2x + 5$
Find the value of shear Force at $x = 2$ m.

(2) what is point of contra-flexture ?

(3) If the shear Force is given by the equation $x^2+2$
Find the value of bending moment at $x = 5$ m.

(4) For a simply supported beam carries a load of 20N and distributed uniformly along the length of 5m. what is the value of maximum B.M ?

(5)

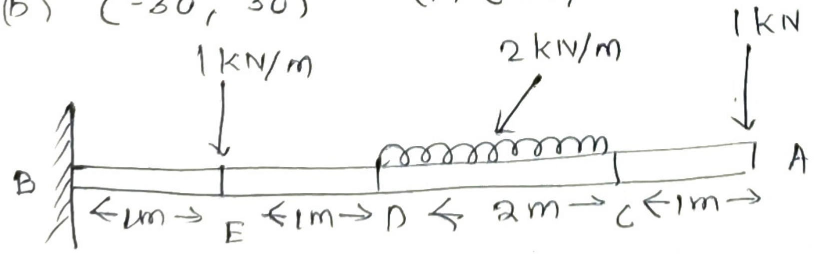



(6) The state of stress at a point in a given body is shown by $\sigma_1 = 100$ MPa, $\sigma_2 = 40$ MPa, Find the value of maximum shear stress

(7) choose the pair of stresses in which the mohr's circle is a point in the figure ?

(a) (-60, 60)
(b) (-30, 30)
(c) (60, 60)
(d) (60, -30)

(8)

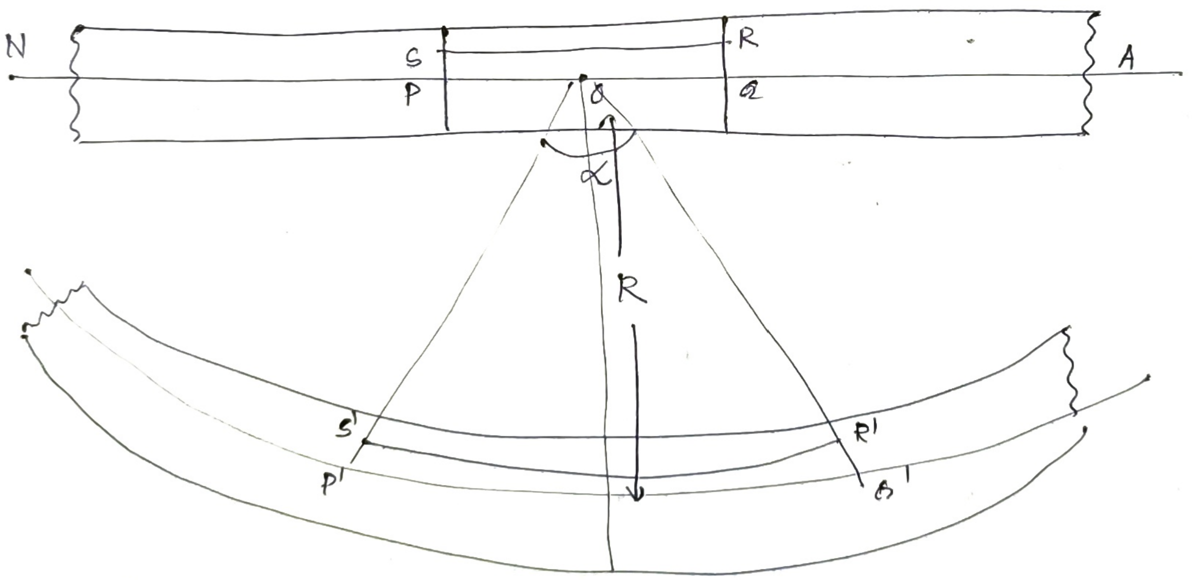



Draw the shear Force and Bending moment diagram ?

## Theory of simple bending

When a beam is loaded it is subjected to bending moment so there is also bending stress acts on the crossponding section. In order to determine the value of the bending stress we have to assume that

(1) The material of the beam is perfectly homogeneous
(2) The stress induced is proportional to strain and at no place that exceed elastic limit
(3) The value of elasticity is same throughout the beam.
(4) The transverse section of the beam which is plane before bending, Remains plane after bending
(5) loads are applied in the plane of bending.
(6) The transverse section of the beam is symmetrical about a line passing through the C.G in the plane of bending.




As a Result of the bending moment the bar becomes a curved shape and looks like arc of a circle. We have seen that outer Radius is in tension and inner part is compress and at some point there will be no stress. This layer is called Neutral layer or Neutral axis.

$$\boxed{\frac{M}{I_{NA}} = \frac{\sigma_b}{Y} = \frac{E}{R}}$$

Y = distance from neutral axis

R = Radius of curvature

This equation is called bending equation

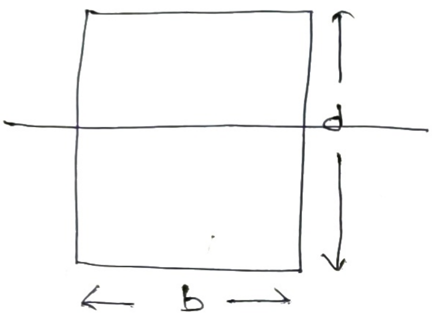
From the equation we have

$$\frac{M}{I_{nx}} = \frac{\sigma_b}{Y} \Rightarrow \sigma_b = \frac{M \times Y}{I_{nx}} = \frac{M}{\frac{I_{nx}}{Y}} = \frac{M}{Z}$$

so $\boxed{Z = \frac{I_{nx}}{Y}}$ $Z$ is called section modulus

Note → The strength of a Beam generally depend upon Section modulus.

(1) Rectangle →

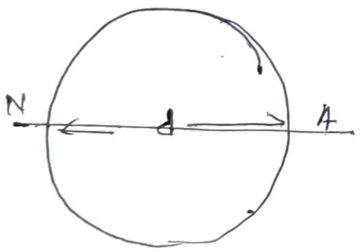


$$\sigma_b = \frac{M}{Z}$$

$$Z = \frac{I_{nx}}{Y} = \frac{\frac{b \times d^3}{12}}{d/2} = \frac{b \times d^2}{6}$$

$$\text{Moment} = \sigma_b \times Z = \sigma_b \times \frac{b \times d^2}{6}$$

(2) circular section →



$$I_{nx} = \frac{\pi d^4}{64}$$

$$Z = \frac{\frac{\pi d^4}{64}}{d/2} = \frac{\pi d^3}{32}$$

For a hollow section $= \frac{\pi}{32} \frac{[D_o^4 - D_i^4]}{D_o}$

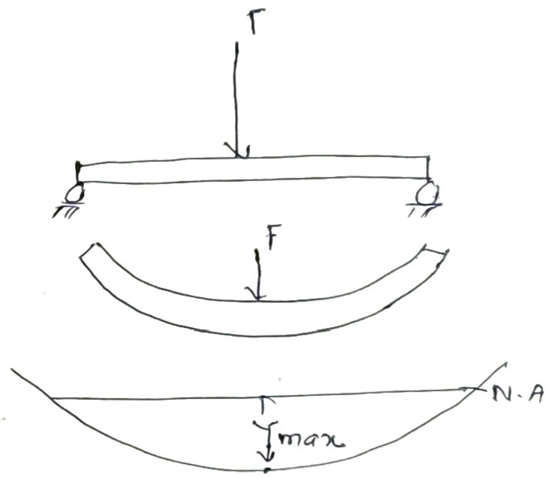
(Q) A 250 mm (depth) × 150 mm (width) Rectangular beam is subjected to maximum B.M of 750 N-m Find the maximum stress in the beam if $E = 2 \times 10^5$ N/m$^2$ Findout the Radius of curvature of the beam section ?

(2) A beam of rectangular section having base width 20 cm and height of 30 c.m is subjected to a shear stress of 3 kN. Find the value of maximum shear stress and the distribution diagram?

(3) A circular beam of 150 mm diameter is subjected to a shear force of 7 kN calculate the value of maximum shear stress and sketch the distribution diagram?

## Deflection

under load the neutral axis becomes a curve line and is called the elastic curve. The deflection (Y) is the vertical distance between the a point on elastic curve and the un-loaded neutral axis.




## Differential Equation of deflection

consider a small portion of the arc and differentiating we get

$$\frac{1}{R} = \frac{d^2Y}{dx^2}$$

As from the bending equation we know

$$\frac{M}{I_{nx}} = \frac{\sigma_b}{Y} = \frac{E}{R} \Rightarrow M = E \times \frac{I_{nx}}{R}$$

$$= E \times I_{nx} \times \frac{d^2Y}{dx^2}$$

Simply we can say $\boxed{M_x = E \times I \times \frac{d^2Y}{dx^2}}$

There are various methods for solving deflection problems

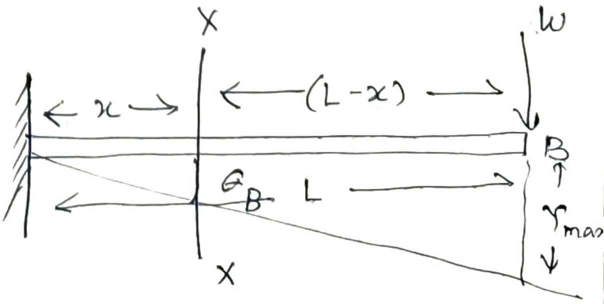
(1) Double integration method

(2) Moment Area method

(3) Macaulay's Method

## Method-1

Let's consider a section X-X which is at a distance of x from the fixed end.



$M_{X-X} = -Wx(L-x)$

As we know the bending equation $M = EI \times \frac{d^2y}{dx^2}$

$$-W(L-x) = EI \times \frac{d^2y}{dx^2}$$

$$\Rightarrow -W \times L + W \times x = EI \times \frac{d^2y}{dx^2}$$

Now integrating Both sides w.r. to x

$$-W\left(L \times x - \frac{x^2}{2}\right) + C_1 = EI \times \frac{dy}{dx}$$

To Remove the constant of integration we use the boundary conditions when $x = 0$ then $\frac{dy}{dx} = 0$, so $C_1 = 0$

$$EI \times \frac{dy}{dx} = -W\left(L \times x - \frac{x^2}{2}\right) \quad \ldots \text{ slope equation}$$

Slope at B we have to take $x = L$

$$EI \times \frac{dy}{dx} = -W\left(L \times L - \frac{L^2}{2}\right)$$

$$\frac{dy}{dx} = \frac{-W \times L^2}{2EI} = \theta_B$$

To get the deflection we have to again integrate it w.r.to x

$$EI \times \frac{dy}{dx} = -W\left(L \times x - \frac{x^2}{2}\right)$$

$$\int EI \times dy = \int -W\left(L \times x - \frac{x^2}{2}\right) dx$$

$$EI \times y = -W\left(L \times \frac{x^2}{2} - \frac{1}{2} \times \frac{x^3}{3}\right) + C_2$$

$$EI \times y = -W\left(\frac{L \times x^2}{2} - \frac{x^3}{6}\right) + C_2$$

again at Fixed end $x = 0,\ y = 0,\ \frac{dy}{dx} = 0$

$$EI \times y = -W\left(\frac{L \times L^2}{2} - \frac{L^3}{6}\right) + 0$$

$$y = -\frac{WL^3}{3EI}$$

so the downward deflection at B $= \frac{WL^3}{3EI}$

## Sign convention

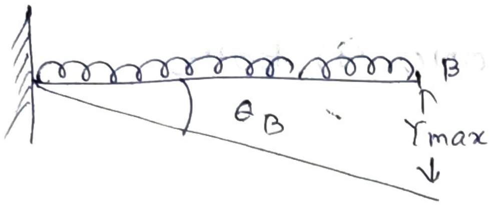
(1) Slope is negative when rotates clockwise.

(2) Deflection is negative when measured downward.

(3) upward deflection is taken as positive.

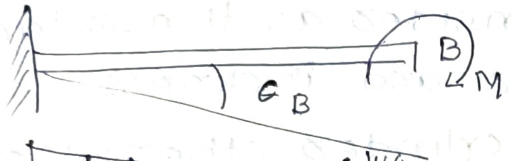
## Some standard Results

(1)



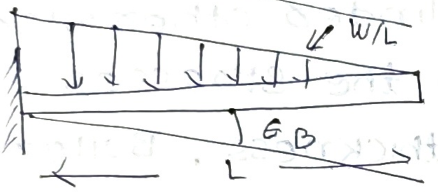

$\theta_B = -\frac{wL^3}{6EI}$ $\quad Y_{max} = -\frac{wL^4}{8EI}$

(2)



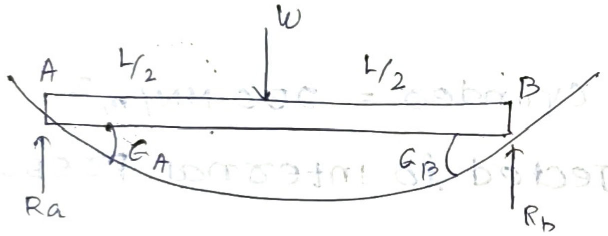

$\theta_B = -\frac{M \times L}{E \times I}$ $\quad Y_{max} = -\frac{ML^2}{2EI}$

(3)




$\theta_B = -\frac{wL^3}{24EI}$ $\quad Y_{max} = -\frac{wL^4}{30EI}$

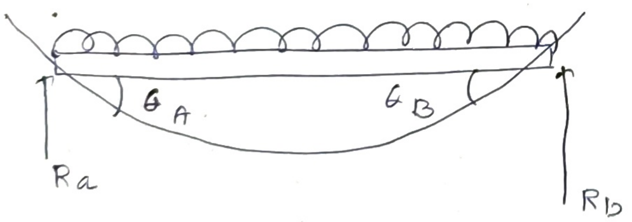
(4)




$\theta_A = -\frac{wL^2}{16EI}$ $\quad Y_{max} = Y_C = -\frac{wL^3}{48EI}$

$\theta_B = \frac{wL^2}{16EI}$

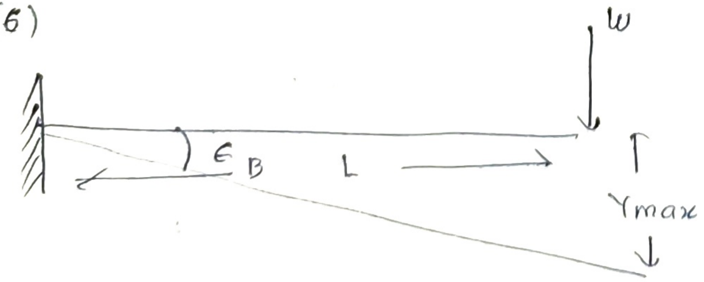
(5)




$\theta_A = -\frac{wL^3}{24EI}$ $\quad Y_{max} = \frac{5wL^4}{384EI}$

$\theta_B = \frac{wL^3}{24EI}$

(6)




$\theta_B = -\frac{wL^2}{2EI}$ $\quad Y_{max} = \frac{wL^3}{3EI}$

## Problems on deflection

(2) A cantilever beam 1.5 m long carries a U.D.L over the entire length. Find the deflection at the free end if the slope at the free end is 1.5°.

(3) The amount of deflection of a cantilever beam is depend upon
(a) cross-section (b) Load applied (c) NONE (d) Both

## Stress Analysis in thin cylinder

A cylindrical vessel may be consider as thin or thick depending upon the Relation of diameter and thickness if $\frac{t}{d} < \frac{1}{20}$ it is considered as thin cylinder otherwise it is considered as thick cylinder. here the stresses are uniformly distibuted over the wall thickness. Boiler tanks, pipe etc.

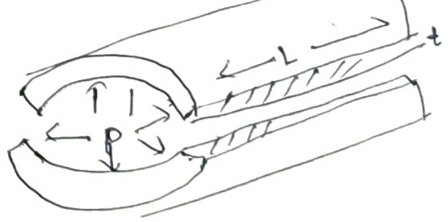
Thin cylinder = 30 MN/m² Thick cylinder = 250 MN/m²

when these thin cylinder are subjected to internal pressure The stresses are →

(1) hoop / circumferential stress → It acts at the tangential directio to the circumference of the shell.

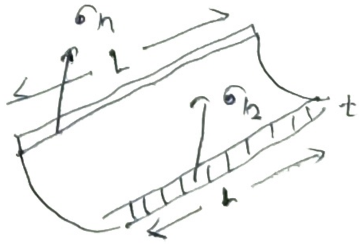
d = internal diameter $t$ = thickness

P = internal pressure $\sigma_c$ = hoop stress



Brusting Force (P) = Resisting Force

$\Rightarrow p \times d \times \cancel{l} = \sigma_c \times 2\cancel{l}t.$



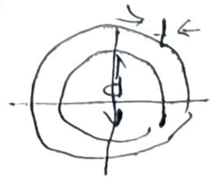
$\Rightarrow \sigma_c = \frac{P \times d}{2 \times t} = \sigma_h$

(2) longitudinal stress → It acts parallel to the longitudinal axis of the shell

Brusting pressure = Resisting Force

$P \times \frac{\pi}{4} \times d^2 = \sigma_L \times \pi d t$

$\sigma_L = \frac{P \times d}{4 \times t}$



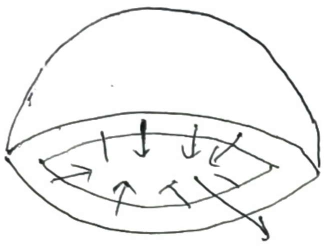
## Maximum shear stress

The maximum $\tau_{max} = \dfrac{\sigma_h - \sigma_L}{2}$

$$= \frac{\frac{p \times d}{2 \times t} - \frac{p \times d}{4 \times t}}{2}$$

$$= \frac{p \times d}{8 \times t}$$

## Stress in Spherical shell



$F = p \times \frac{\pi}{4} \times d^2$

d = diameter of the shell
t = thickness
p = intensity of pressure
$\sigma_c$ = hoop stress

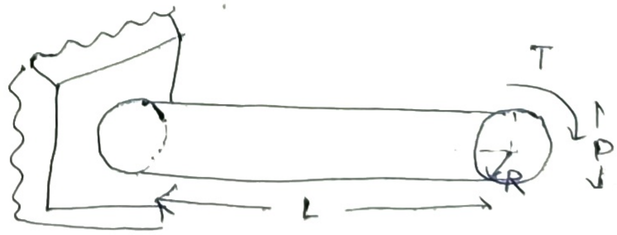
$$p \times \frac{\pi}{4} \times d^2 = \pi d \times t \times \sigma_h$$

$$\Rightarrow \sigma_h = \frac{p \times d}{4 \times t}$$

## Torsion of Circular shaft

To transmite the energy by rotation it is neccary to apply a turning Force. If this Force multipiyed by the Radius of the shaft is called twisting moment and due to this twisting moment the Rotation occours is called torsion.

## Torsion equation




### Assumption

(1) material of the shaft is homogenous throughout
(2) The shaft is circular in crossection and remains circular after loading.
(3) The twist along the shaft is uniform throughout.
(4) shear stress produced does not exceed propeortional limit

The torsion equation can be written as

$$\frac{T}{I_p} = \frac{\tau}{R} = \frac{G \times \theta}{L}$$

T = Torque / Twisting moment
$I_p$ = polar modulus

$\tau$ = Shear stress
G = modulus of Rigidity
$\theta$ = angle of twist

$$\frac{T}{I_p} = \frac{\tau}{R}$$

$$\Rightarrow T = \frac{\tau}{R} \times I_p$$

$$= \tau \times \frac{I_p}{R}$$

since For a shaft $I_p$ and R values are constant it is taken as polar modulus $\boxed{Z_p = \frac{I_p}{R}}$

Thus from the equation we oberserve that the polar modulus is measure of strength of shaft in torsion.

## Torsional Rigidity

$$\frac{T}{I_p} = \frac{G \times \theta}{L}$$

$$\Rightarrow \theta = \frac{T \times L}{G \times I_p}$$

For a given shaft we know that G, L, $I_p$ are constant. in such cases angle of twist $\propto$ Twisting moment

so $\frac{G \times I_p}{L} = K$ = torsional Rigidity

## Power transmitted by shaft

$$P = \frac{2\pi N T}{60,000} \text{ kW}$$

N = Revolution speed.

T = Torque = $F \times r$

## comparision of solid and hollow shaft

(a) comparison by strength →

we have to assume that shaft are having same length, same material and same weight and same shear stress

$$T_{solid} = \frac{\pi}{16} \times \tau \times D^3$$

$$T_{hollow} = \frac{\pi}{16} \times \tau \times \left[ \frac{D_H^4 - d_H^4}{D_H^4} \right]$$

$$\frac{\text{strength of hollow}}{\text{strength of solid}} = \frac{\cancel{\frac{\pi}{16}} \times \cancel{\tau} \times \left[ \frac{D_H^4 - d_H^4}{D_H^4} \right]}{\cancel{\frac{\pi}{16}} \times \cancel{\tau} \times D_S^3} = \frac{D_H^4 - d_H^4}{D_H \cdot D_S^3}$$

$$\frac{T_H}{T_S} = 1.44$$

This shows that torque transmitted by the hollow shaft is greater than solid shaft.

(b) comparision by weight → shafts are having same length, same material and torque applied are also same

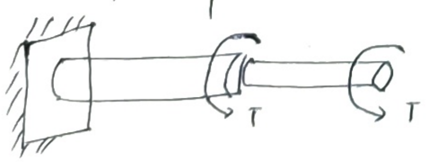
$$\frac{\text{weight}_{hollow}}{\text{weight}_{solid}} = \frac{W_H}{W_S} = \frac{A_H}{A_S} = \frac{\cancel{\frac{\pi}{4}} \times (D_H^2 - d_H^2)}{\cancel{\frac{\pi}{4}} \times D_S^2} = \frac{D_H^2 - d_H^2}{D_S^2}$$

$$\frac{W_H}{W_S} = 0.7829 \Rightarrow W_H = 0.7829 \times W_S$$

we found that the hollow shaft is lighter than solid shaft.

## shaft in series

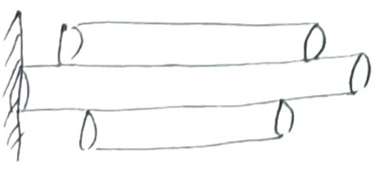
in order to form a composite shaft sometimes two shafts are connected in series in such cases they transmite same torque.




$$\theta = \theta_1 + \theta_2$$

$$= \frac{T_1 \times L_1}{G_1 \times I_{P_1}} + \frac{T_2 \times L_2}{G_2 \times I_{P_2}}$$

## Shaft in Parallel

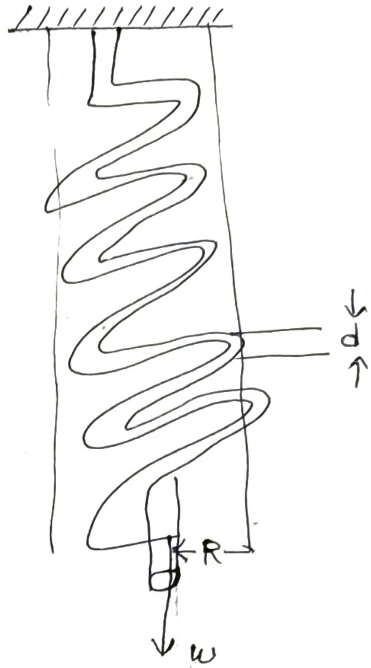


in such cases angle of twist is same

$$\theta = \theta_1 = \theta_2 = \theta_3$$

$$\frac{T_1 \times L_1}{G_1 \times I_{P_1}} = \frac{T_2 \times L_2}{G_2 \times I_{P_2}}$$

## Closed coiled helical springs




$R$ = Radius of the coil

$d$ = diameter of the coil

$\delta$ = deflection

$G$ = modulus of Rigidity

$n$ = no. of turns of the coil

$\theta$ = angle of twist

$\tau$ = shear stress

$I_p$ = polar modulus

Length of wire $= 2\pi R n$

according to torsion equation

$$\frac{T}{I_p} = \frac{G \times \theta}{L} = \frac{\tau}{R}$$

$$\Rightarrow \quad T = \frac{\tau \times I_p}{R} = \frac{\tau \times \pi d^4}{32 \times \frac{d}{2}} = \frac{\pi}{16} \times \tau \times d^3$$

$$\Rightarrow \quad \tau = \frac{16T}{\pi d^3} = \frac{16 \times W \times R}{\pi d^3}$$

Again $\frac{T}{I_p} = \frac{G \times \theta}{L} \Rightarrow \theta = \frac{L \times T}{I_p \times G} = \frac{L \times W \times R}{G \times \frac{\pi d^4}{32}} = \frac{32 \times L \times W \times d/2}{G \times \pi d^4}$

$$= \frac{64 W R^2 n}{G d^4}$$

Deflection of a spring $\delta = (R \times \theta)$

$$= \frac{R \times 64 W R^2 n}{G \times d^4} = \frac{64 W R^3 n}{G d^4}$$

Stifness of the spring $K = \frac{W}{\delta} = \frac{\cancel{W}}{\frac{64 \cancel{W} R^3 n}{G d^4}} = \frac{G d^4}{64 R^3 n}$

Springs are the elastic member which disorts under the application of load. It is widely used in Railway, carrige, motor cars etc.

## Strain energy

When an elastic body is loaded it underoges some deformation and after the removal of load it regains its original shape during this period the energy stores inside the body is called strain energy.

The strain energy stroes inside the body within the elastic limit is called Resilience.

maximum energy stroed inside the body within the elastic limit is called proof Resilience.

Proof Resilience per unit volume is called modulus of Resilence

For a material with load P

$$\text{strain energy } (U) = \frac{\sigma^2 V}{2AE} = \frac{\sigma^2 \times V}{2E} = \frac{\tau^2}{4G} \times \text{volume}$$

$$\text{proof Resilence } (U_p) = \frac{\sigma_p^2}{2E} \times V$$

$$\text{modulus of Resilience} = \frac{\sigma_p^2}{2E}$$

## Theory of column

A column is a long vertical slender bar or vertical member subjected to an axail compressive load.

column having length less than 8 times of their diameter is called short column

column having their length more than 30 times of their Respective diameter is called long column.

## Assumption

(1) column is initially straight and uniform dimension.
(2) compressive load is axail.
(3) material of the column is homogenous and isotropic.
(4) weight of column is neglected.
(5) limit of proportionally does not exceed.