

1ST SEM ./COMMON / 2022(W)

Th-3 Engineering Mathematics-I

Full Marks: 80

Time- 3 Hrs

Answer any five Questions including Q No.1& 2
Figures in the right hand margin indicates marks

1. Answer **All** questions 2 x 10

a. Find the value of $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$.

b. Find x and y when $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$.

c. Find the minimum and maximum value of $5 \sin x + 12 \cos x$.

d. Find $\tan\left(\frac{\pi}{4} + 2 \cot^{-1} 3\right)$.

e. Determine the ratio in which the line segment joining $(2, -3)$ and $(5, 6)$ is divided by x -axis.

f. Find the perpendicular distance from the point $(2, 1)$ to the straight line $12x - 5y + 9 = 0$.

g. Find the equation of the circle which touches the x -axis and whose centre is at the point $(3, 4)$.

h. Find image of the point $(1, -2, 4)$ with respect to YZ -plane.

i. Find the direction cosines of a straight line whose direction ratios are 1, 2, 3.

j. Find the centre and radius of the sphere
 $3x^2 + 3y^2 + 3z^2 - 12x - 6y + 9z + 1 = 0$.

2. Answer **Any Six** Questions 6 x 5

a. Without expanding prove that

$$\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$$

b. Solve the following equations by Matrix Method,
 $x + 2y = 3$ and $3x + y = 4$

- c. Prove that $\sin 10^\circ \cdot \sin 30^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ = \frac{1}{16}$
- d. Find the equation of the straight line which passes through the point (3, 4) and sum of its intercepts on the axes is 14.
- e. Find the equation of plane passing through the point (2, -2, -1) and parallel to the plane $2x + y - 3z - 2 = 0$.
- f. Find the equation of the sphere whose centre at (3, 1, -2) and the sphere passing through the point (1, 1, 2).
- g. If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, show that $x + y + z = xyz$.

3 a. Solve the following equations by Cramer's Rule, 5
 $2x - 3y + 5 = 0$ and $5y - 3x - 8 = 0$

- b. Find the equation of the plane passing through the intersection of planes $2x + 3y - 4z + 1 = 0$ and $3x - y + z + 2 = 0$, and passing through the point (3, 2, 1). 5

4 a. Find the equation of the circle which passes through the points (1, -2) and (4, -3) and has its centre lies on the line $3x + 4y = 7$. 7

- b. If the point (x, y), (1, -2) and (3, -4) are collinear, 3
 prove that $x + y + 1 = 0$.

5 a. Find the equation of the sphere passing through (1, 2, -3) and (3, -1, 2) and centre lying on X-axis. 5

- b. If $A + B + C = \pi$, 5
 Prove that $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$.

6 a. In a ΔABC if $m\angle A = 90^\circ$, prove that $\tan^{-1} \frac{b}{a+c} + \tan^{-1} \frac{c}{a+b} = \frac{\pi}{4}$, 5
 where a , b and c are the sides of the triangle.

b. Verify that $[AB]^T = B^T A^T$, 5
 where $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$.

7 a. Find the equation of a straight line parallel to the line $2x + 3y + 11 = 0$ and sum of its intercepts on the axes is 15. 6

- b. If $A + B = 45^\circ$, show that $(1 + \tan A)(1 + \tan B) = 2$. 4