

1st SEMESTER/COMMON/2021(W)(NEW)
Th3 ENGINEERING MATHEMATICS - I

Full Marks: 80

Time- 3 Hrs

Answer any five Questions including Q No.1& 2
Figures in the right hand margin indicates marks

1. Answer all questions 2 x 10
- a. Find M_{23} and C_{32} of the determinant $\begin{vmatrix} 4 & 3 & 8 \\ 6 & 7 & 5 \\ 9 & 0 & 6 \end{vmatrix}$.
- b. Find k for which the following lines are perpendicular to each other
 $2x+3y-1=0$ and $kx-4y+2=0$.
- c. Find $\sin(\tan^{-1} x + \cot^{-1} x)$.
- d. Find the centre and radius of the sphere
 $(x - 2)(x + 2) + y^2 + (z - 3)(z + 3) = 0$
- e. If $\begin{bmatrix} 3 & 4 & 2 \end{bmatrix} \times B = \begin{bmatrix} 2 & 1 & 0 & 3 & 6 \end{bmatrix}$. Find order of B
- f. What is ASTC Rule in Trigonometry?
- g. If the equation $3x^2 - \frac{k}{2}y^2 - 6x + 9y - 3 = 0$ represents a circle, find k.
- h. A line makes angle α, β, γ with X,Y,Z axes, then find
 $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.
- i. Find the multiplicative inverse of the matrix $\begin{pmatrix} 4 & 3 \\ 5 & 4 \end{pmatrix}$.
- j. Find the intercepts cut off by the plane $2x+3y-z=6$ on the axes.
2. Answer **Any Six** Questions 5X6
- a. Find the angle between two lines whose direction ratios are $\langle 1,2,1 \rangle$ and $\langle 2, -3,4 \rangle$.
- b. Find the equation of the circle whose diameter is the portion of the line $3x+4y-12=0$ intercepted between the coordinate axes.
- c. Prove without expanding
$$\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = abc(a - b)(b - c)(c - a).$$
- d. Find the maximum and minimum value of the following
 $6 \cos x - 8 \sin x - 3$
- e. Find the equation of the line which passes through $(-3, 7)$ and makes intercepts on the axes equal in magnitude but opposite in sign.

- f. In a triangle ABC if $m\angle A = 90^\circ$, prove that
 $\tan^{-1} \frac{b}{a+c} + \tan^{-1} \frac{c}{a+b} = \frac{\pi}{4}$, where a, b, c are sides of the triangle.
- g. If $A = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$, evaluate $A^2 - 6A + 8I$, where I is the Identity matrix of the given order.

Answer any **three** questions

- 3 a. Find the equation of the line passing through intersection of the lines $x + 3y - 7 = 0$ and $3x - y - 11 = 0$ and centroid of the triangle whose vertices are the points (3,-1), (1, 3) and (2, 4). 5
- b. Evaluate $\sin 18^\circ$. 5
- 4 a. Find the equation of the plane passing through the point (-1, 3, 2) and perpendicular to the planes $x+2y+2z=5$ and $3x+3y+2z=8$. 7
- b. Solve by Cramer's Rule
 $2x+3y=1$ and $-x+y=-3$ 3
- 5 If $A + B + C = \pi$, prove that
 $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2 \cos A \cos B \cos C$ 10
- 6 Find the equation of a sphere whose centre lies on the plane $x + y + z = 0$ and which passes through the points (1,-3,4), (1,-5,2) and (1,-3,0). 10
- 7 a. Evaluate $\tan^{-1} \left[\frac{\sqrt{1-\sin x} + \sqrt{1+\sin x}}{\sqrt{1-\sin x} - \sqrt{1+\sin x}} \right]$ 6
- b. Find the value of 'a' so that the points (1, 4), (2,7), (3,a) are collinear. 4