Ist -SEM /COMMON/2019(W)/ (New)

Th. 3-ENGG. MATHEMATICS-1

Full Marks: 80 Time : 3 Hours.

Answer any FIVE Questions including Q No. 1& 2.

Figures in the right hand margin indicates marks.

1. Answer ALL the questions. **(2x10)**

(i) Find all the minors of the determinant $\begin{vmatrix} 3 & -4 \\ 8 & -1 \end{vmatrix}$.

(ii) Find the maximum value of the determinant $\begin{vmatrix} \cos x & \sin x \\ \sin x & \cos x \end{vmatrix}$.

(iii) Prove that $4\cos 105^0.\cos 15^0 + 1 = 0$

(iv) Find the value of $\frac{\tan 15^0}{1+\tan^2 15^0}$

(v) Find the value of $\sec^2(\tan^{-1}2) + cosec^2(\cot^{-1}3)$

(vi) Find the slope and y-intercept of the line $2x - 3y + 8 = 0$

(vii) Find the center and radius of the circle $x^2 + y^2 - 8x + 2y - 6 = 0$

viii) Find the value of k such that the three points $(1, -2, 3), (3, -1, 2)$ and $(7, 1, k)$ are collinear.

(ix) Write the normal form of equation of plane.

(x) Find the distance between the points (2,3,5) and (4,3, − 1).

2. Answer any SIX of the following. **(5X6)**

(i) Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

(ii) Verify $(AB)^T = B^T A^T$ where $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{pmatrix}$

(iii) Show that the equation $\sin\theta = a + \frac{1}{a}$ doesnot have a solution for every real number $a \neq 0$.

(iv) If $\theta = \alpha + \beta$ and $\tan\alpha : \tan\beta = x : y$, then prove that $\sin(\alpha - \beta) = \frac{x-y}{x+y}\sin\theta$.

(v) Find the coordinates of the points which divide internally and externally the line joining (1, − 3) and (− 3,9) in the ratio 1:3.

(vi) Reduce $x + \sqrt{3}y + 8 = 0$ to normal form of equation of straight line.

(vii) Find the equation of the plane which passes through the point (3,4,-1) and is parallel to the plane 2x-3y+5z+7=0. Also calculate the distance between them.

3.	If A+B+C= π , then prove that SinA + SinB - SinC = 4 sinA/2 Sin B/2 Cos C/2	10
4.	Prove that $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$	10
5.	Prove that $cos20^0.cos40^0.cos60^0.cos80^0 = \frac{1}{16}$	10
6.	Find the equation of the line passing through the point of intersection of 2x-y-1=0 and 3x-4y+6=0 and perpendicular to the line x+y+2=0.	10
7.	Find the equation of the sphere which passes through the points (0,0,0), (0,1,0),(1,0,0) and (0,0,1)	10

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