ENGINEERING MATHEMATICS - 1

(Theory — 3)

Full Marks : 80

Time : 3 hours

Answer any four questions including **Q. Nos. 1 & 2**

Figures in the right-hand margin indicate marks

GROUP — A

1. Answer *all* : 2×10

    (a) Find the value of

    $$\frac{\cos 15^\circ + \sin 15^\circ}{\cos 15^\circ - \sin 15^\circ}$$

    (b) Find the minimum value of

    $$5\sin x + 12\cos x$$

    (c) Solve by Crammer's rule

    $$x + y = 2$$
    $$2x - y = 1$$

    (d) Find the ratio in which the line segment joining $(-2, -3)$ and $(4, 5)$ is devided by the X-axis.

    (e) Find the centre and radius of the circle

    $$2x^2 + 2y^2 - 4x + 6y + 2 = 0$$

    (f) If $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 1 \\ 5 & 2 \end{pmatrix}$

    (g) Find the image of the point $(-2, 3, 1)$ with respect to xy-plane.

    (h) Find the distance of the point $P(3, 4, 5)$ from X-axis.

    (i) Find length of perpendicular of the point $(1, -1, -1)$ from the plane

    $$2x + y + 2z + 4 = 0.$$

    (j) Find equation of the sphere which has centre $(1, 2, 3)$ and passes through $(0, 0, 0)$.

(Turn Over)

GROUP – B

2. Answer any *six* questions :   5×6

   (a) Prove that

   $$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

   (b) Find adjoint of the matrix

   $$\begin{pmatrix} 2 & 3 \\ 4 & 2 \end{pmatrix}$$

   (c) If $A + B + C = \pi$ then prove that

   $$\sin 2A + \sin 2B + \sin 2C = 4\sin A \cdot \sin B \cdot \sin C$$

   (d) Find equation of the line which passes through the point (1, 2) and perpendicular to the line $4x + 3y + 5 = 0$.

   (e) Find equation of the circle which passes through (0, 0) and has intercepts a and b on X-axis and Y-axis respectively.

   (f) Find equation of the plane bisecting the line segment joining the points (−1, 4, 3) and (7, −2, 1) at right angle.

   (g) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi/2$ then prove that $xy + yz + zx = 1$.

   (h) If $\dfrac{1}{a+c} + \dfrac{1}{b+c} = \dfrac{3}{a+b+c}$ prove that $\underline{|C} = 60°$

GROUP – C

Answer any **three** questions :   10 × 3

3. (a) Find angle between two planes

   $$x + 2y + 2z - 7 = 0$$
   $$2x - y + z - 6 = 0$$

   (b) Find equation of the sphere which passes through (0, 0, 0), (a, 0, 0), (0, b, 0) and (0, 0, c).

4. (a) Find inverse of the matrix

   $$\begin{pmatrix} 4 & 6 \\ 2 & 4 \end{pmatrix}$$

(b) Find equation of the line passing through intersection of $2x - y - 1 = 0$ and $3x + 4y + 6 = 0$ and passes through $(0, 0)$.

5. (a) Prove that

$$2\cos\frac{\pi}{16} = \sqrt{2 + \sqrt{2 + \sqrt{2}}}$$

(b) Solve by Crammer's rule

$$2x + y + 2z = 2$$
$$3x + 2y + z = 2$$
$$-x + y + 3z = 6$$

6. (a) Find the foot of perpendicular drawn from the point $(0, 0, 0)$ on the plane

$$2x + y + z - 3 = 0$$

(b) Prove that

$$\sin^2(\sin^{-1} x + \sin^{-1} y + \sin^{-1} z) = \cos^2(\cos^{-1} x + \cos^{-1} y + \cos^{-1} z)$$

7. (a) Find distance of the point $(1, 2)$ from the $x + 2y + 1 = 0$ measured parallel to the line

$$3x - y + 1 = 0.$$

(b) Find equation of the circle which has centre on X-axis and the circle passes through $(0, 0)$ and $(4, 2)$.

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