2nd. SEM. /COMMON/ 2022(S)

TH-3 -ENGINEERING MATHEMATICS -II

Full Marks: 80   Time- 3 Hrs

Answer any five Questions including Q No.1& 2

Figures in the right hand margin indicates marks

1.   Answer **All** questions   2 x 10

1.   a.   Define Modulus Function and represent it graphically.

     b.   Evaluate $\lim_{x \to 0} \frac{x}{\sqrt{1+x} - \sqrt{1-x}}$

     c.   Differentiate $\sec^{-1}\left(\frac{\sqrt{a^2+x^2}}{a}\right)$ with respect to x.

     d.   Define unit vector and find the unit vector of the given vector $2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$.

     e.   *Evaluate the integral* $\int \left(e^{5\ln x} - e^{4\ln x}\right) dx$.

     f.   Define Homogeneous Function and State Euler's Theorem.

     g.   Find the value of α so that $\vec{a} = \hat{\imath} + \hat{\jmath} + \alpha\hat{k}, \vec{b} = 4\hat{\imath} - 3\hat{k}$ are perpendicular to each other.

     h.   Find the order and degree of the following differential equation
          $$\frac{d^2y}{dx^2} = \frac{3y + \frac{dy}{dx}}{\sqrt{\frac{d^2y}{dx^2}}}$$

     i.   *Find the value of* $\int_{-2}^{2} |x|\, dx$.

     j.   If $y = t^2$ and $x = t^3$ find $\frac{dy}{dx}$ at $t = 1$.

                                                                                    6 x 5

2.   Answer **Any six** questions:

     a.   If $f(x) = \begin{cases} ax^2 + b, & if\ x < 1 \\ 1, & if\ x = 1 \\ 2ax - b, & if\ x > 1 \end{cases}$ is continuous at $x = 1$, then find the value of 'a' and 'b'.

     b.   Find $\frac{dy}{dx}$ if $y = (\ln x)^{tanx}$.

     c.   Determine the area within the curve $y^2 = 4ax$ and the x-axis, the ordinate x=4.

     d.   Evaluate $\int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx$.

     e.   Solve $(1 + x^2)dy + (1 + y^2)dx = 0$.

1

f. Find the scalar and vector projections of the vector $2\hat{\imath} - 3\hat{\jmath} - 6\hat{k}$ on the line joining the points (3,4,-2) and (5,6,-3).

g Find $\frac{dy}{dx}$ if $x = \frac{2t}{1+t^2}, y = \frac{2t}{1-t^2}$.

3 i If $\sqrt{1-x^6} + \sqrt{1-y^6} = k(x^3 - y^3)$, prove that

$$\frac{dy}{dx} = \frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$$

7

ii Evaluate $\lim_{x\to 0} \frac{1-cos^3x}{xsin2x}$.

3

4 i If $u = \tan^{-1}(x^2 + y^2 + z^2)$, show that $xu_x + yu_y + zu_z = sin2u$

7

ii If sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

3

5 i Evaluate $\int \frac{2x+11}{\sqrt{x^2+10x+29}} dx$.

6

ii If $y = \tan^{-1} x$, prove that $(1 + x^2)y_2 + 2xy_1 = 0$

4

6 i Solve the following differential equation

$(1 + y^2)dx = (\tan^{-1} y - x)dy$

7

ii Find the derivative of $y = e^x$ by first principle.

3

7 i In a triangle AOB, angle AOB=90^0. If P, Q are the points of trisection of $\overline{AB}$, prove that $OP^2 + OQ^2 = \frac{5}{9}AB^2$ by vector method.

6

ii Evaluate $\int e^x \left(\frac{1}{x^2} - \frac{2}{x^3}\right) dx$.

4