

ENGG. MATH - II

(Theory : 3)

Full Marks : 80

Time : 3 hours

Answer any five questions including Q. Nos. 1 & 2

Figures in the right-hand margin indicate marks

1. Answer all questions :

2 × 10

(a) Evaluate :

$$\lim_{x \rightarrow 1} \left(\frac{\frac{1}{x^2} - \frac{1}{4}}{x - 2} \right)$$

(b) If $u = t^2$ and $v = \sin t^2$, then find $\frac{dv}{du}$.(c) If $f(x, y) = e^{xy}$, then find $y \cdot \frac{\partial f}{\partial y}$.(d) Find derivative of \sqrt{x} w.r.t. x^2 .

(e) Examine the existence of

$$\lim_{x \rightarrow \frac{5}{2}} [x]$$

(f) If $y = c_1 e^x + c_2 e^{-x}$, then find $\frac{d^2 y}{dx^2}$.

(g) Evaluate

$$\int e^{(5x+3)} \cdot dx$$

(h) The two forces act on a particle at a point. Find their resultant if they are $(4\hat{i} + \hat{j} - 3\hat{k})$ and $(3\hat{i} + \hat{j} - \hat{k})$.

(i) Solve

$$\frac{dy}{dx} = \frac{x}{y}$$

(j) Find the derivative of $\sin^{-1}(3x)$.

2. Answer any six questions :

5 × 6

(a) If $f(x) = \begin{cases} \frac{x - |x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$ at $x = 0$.

Show that $\lim_{x \rightarrow 0} f(x)$ does not exist.

(Turn Over)

(b) Evaluate

$$\lim_{x \rightarrow 0} \left(\frac{x - x \cos 2x}{\sin^3 2x} \right).$$

(c) If $y = \tan^{-1}x$, prove that

$$(1 + x^2) y_2 + 2xy_1 = 0.$$

(d) If $f(x, y) = \frac{2x - 3y}{x^2 + y^2}$, find $f_x(1, 2)$ and $f_y(1, 2)$.

(e) Solve the differential equation,

$$x(1 + y^2) dx + y(1 + x^2) dy = 0.$$

(f) Evaluate

$$\int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx.$$

(g) Find the area bounded by the curve $xy = c^2$, the x -axis and $x = 2, x = 3$.

(h) Evaluate

$$\int_0^{\pi/2} \frac{dx}{1 + \cot x}.$$

3. Find the value of 'a' if

$$\lim_{x \rightarrow 2} \frac{\log_e(2x - 3)}{a(x - 2)} = 1. \quad 10$$

4. Differentiate, $\tan^{-1}(\sec x + \tan x)$. 10

5. Evaluate

$$\int \log(1 + x^2) dx. \quad 10$$

6. If $y = (\sin^{-1}x)^2$, show that

$$(1 - x^2) y_2 - xy_1 - 2 = 0. \quad 10$$

7. Find sine of the angle between the vectors \vec{a} and \vec{b} where

$$\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k} \quad \text{and} \quad \vec{b} = \hat{i} + 3\hat{j} + 2\hat{k}. \quad 10$$