3RD SEM./ AE & IE/ ELECTRICAL &MECH/ ELECTRICAL & ETC/ EE(I&C) /
EE[PT]/ ELECTRONICS & COMM/ ELECTRICAL /E&TC / 2022(W)
Th-1   ENGINEERING MATHEMATICS-III

Full Marks: 80   Time- 3 Hrs

Answer any five Questions including Q No.1& 2
Figures in the right hand margin indicates marks

1.   Answer **All** questions 2×10

a.   Find the modulus and amplitude of the complex number $1 + i\sqrt{3}$.

b.   Define the rank of a matrix. Find the rank of the matrix $\begin{pmatrix} 1 & 3 \\ 3 & 9 \end{pmatrix}$.

c.   Find the complementary function of $(D^2 + 2D - 15)y = \sin 3x$.

d.   Construct a partial differential equation for the function $z = f(x^2 - y^2)$.

e.   Define Laplace transform of a function and find $\mathcal{L}(1)$.

f.   Define periodic function and find the period of the function $\cos(2x + 5)$.

g.   Find a_0 in the Fourier series of $f(x) = x$ in the interval $[-\pi, \pi]$.

h.   Define the Shift operator and write the relation between shift operator (E) and forward difference operator (Δ).

i.   Calculate $\Delta \tan^{-1} x$ by taking the interval difference as unity.

j.   State Lagrange's interpolation formula to interpolate a function.

2.   Answer **Any Six** Questions 5×6

a   If $x + \dfrac{1}{x} = 2 \cos \theta$, Show that $x^n + \dfrac{1}{x^n} = 2 \cos n\theta$.

b.   Determine the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ by using elementary row transformation method.

c.   Solve $(D^2 + 4D + 3)y = e^{-x} \sin x$.

d.   Find the Laplace transform of the function $f(t) = \begin{cases} 3, & 0 \le t < 2 \\ -1, & 2 \le t < 4 \\ 0, & t \ge 4 \end{cases}$

e.   Find the Fourier series of the function $(x) = \begin{cases} 0, & 0 < x < \pi \\ 1, & \pi < x < 2\pi \end{cases}$.

f.   Find a real root of the equation $x^3 - 2x - 5 = 0$,
      by using bisection method in three steps.

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g. Find the cubic polynomial which takes the following values:

x	:	0	1	2	3
$f(x)$	:	1	2	1	10

Hence, evaluate $f(4)$.

3 a. For what values of λ and μ the equations   7

$2x + 3y + 5z = 9, \quad 7x + 3y - 2z = 8, \quad 2x + 3y + \lambda z = \mu,$

have (i) no solution, (ii) a unique solution and (iii) an infinite number of solutions.

b. If $1, \ \omega, \ \omega^2$ are the three cube roots of unity,

Prove that $(1 - \omega + \omega^2)^5 + (1 + \omega - \omega^2)^5 = 32$ 3

4 a. Solve $x(y - z)p + y(z - x)q = z(x - y)$.   7

b. Find the inverse Laplace transform of $F(s) = \left(\frac{4s - 3}{s^2 + 9}\right)$ 3

5 a. Find $\mathcal{L}(t \sin^2 t)$ 5

b. Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ by using Simpson's 1/3 rule,

and compare the result with its actual value.   5

6 Obtain the Fourier series for

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \frac{\pi x}{4}, & 0 < x < \pi \end{cases}.$$ 10

Hence Prove that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots \ldots \ldots$

7 a. Develop a recurrence formula for finding the value of $\sqrt{10}$, using Newton-Raphson   7
method and hence, compute this value correct to three decimal places.

b. Find the square roots of $-5 + 12\sqrt{-1}$.   3