GOVERNORS AND FLYWHEEL

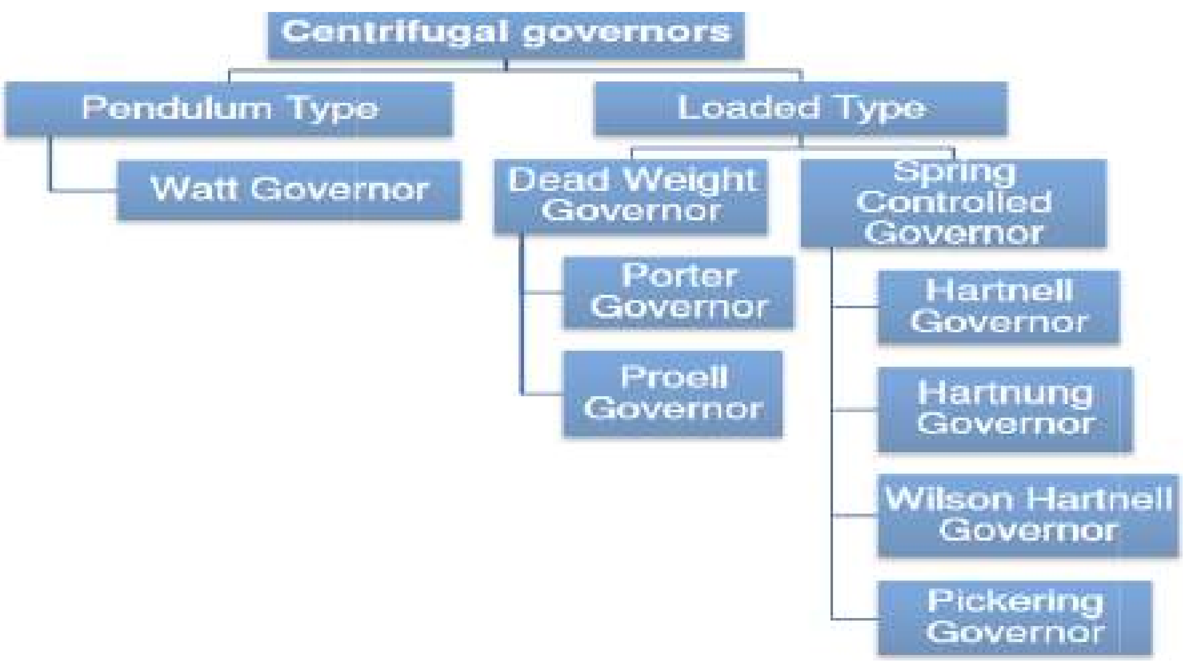
Governor:

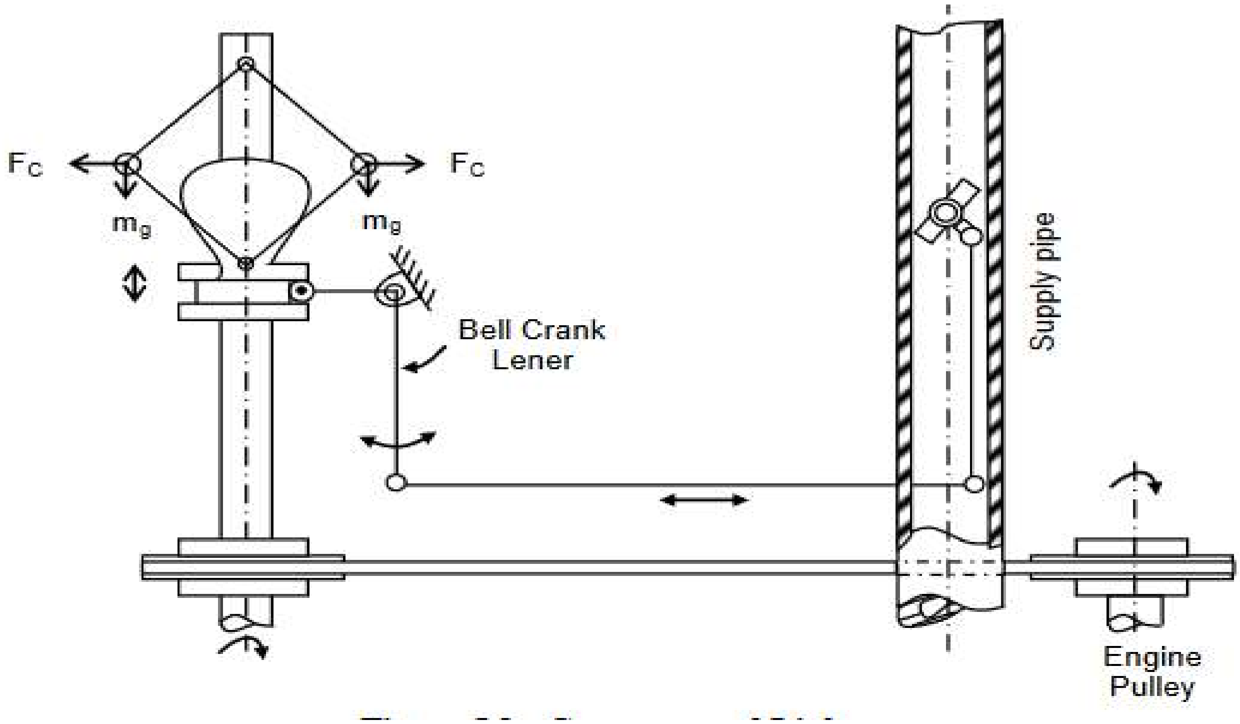
To minimize fluctuations in the mean speed which may occur due to load variation, governor is used.

The function of governor is to increase the supply of working fluid going to the prime-mover when the load on the prime-mover increases and to decrease the supply when the load decreases so as to keep the speed of the prime-mover almost constant at different loads.

Example: when the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid. On the other hand, when the load on the engine decreases, its speed increases and hence less working fluid is required.

Classification of Centrifugal Governors:





Centrifugal Governor:

In these governors, the change in centrifugal forces of the rotating masses due to change in the speed of the engine is utilized for movement of the governor sleeve. These governors are commonly used because of simplicity in operation.

- It consists of two balls of equal mass, which are attached to the arms. These balls are known as governor balls or fly balls.
- The balls revolve with a spindle, which is driven by the engine through bevel gears.
- The upper ends of the arms are pivoted to the spindle, so that the balls may rise up or fall down as they revolve about the vertical axis.
- The sleeve revolves with the spindle; but can slide up and down. The balls and the sleeve rise when the spindle speed increases, and falls when the speed decreases.
- In order to limit the travel of the sleeve in upward and downward directions, two stops S, S are provided on the spindle.
- The sleeve is connected by a bell crank lever to a throttle valve. The supply of the working fluid decreases when the sleeve rises and increases when it falls.
- When the load on the engine increases, the engine and the governor speed decreases. This results in the decrease of centrifugal force on the balls. Hence the balls move inwards and the sleeve moves downwards. The downward movement of the sleeve operates a throttle valve at the other end of the bell crank lever to increase the supply of working fluid and

thus the engine speed is increased. Hence, the extra power output is provided to balance the increased load.

- When the load on the engine decreases, the engine and the governor speed increases, which results in the increase of centrifugal force on the balls. Thus the balls move outwards and the sleeve rises upwards. This upward movement of the sleeve reduces the supply of the working fluid and hence the speed is decreased. Hence, the power output is reduced.

Types of Centrifugal Governors: Depending on the construction these governors are of two types: (a)Gravity controlled centrifugal governors, and(b)Spring controlled centrifugal governors.

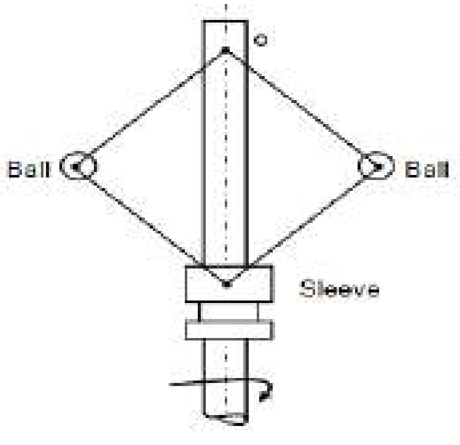
Gravity Controlled Centrifugal Governors-In this type of governors there is gravity force due to weight on the sleeve or weight of sleeve itself which controls movement of the sleeve. These governors are comparatively larger in size.

Spring Controlled Centrifugal Governors-In these governors, a helical spring or several springs are utilized to control the movement of sleeve or balls. These governors are comparatively smaller in size.

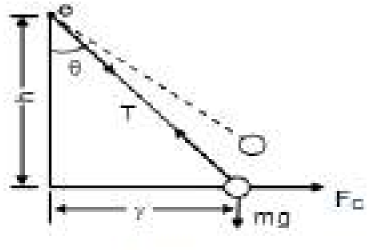
There are three commonly used gravity controlled centrifugal governors :(a)Watt governor(b)Porter governor(c) Proell governor Watt governor does not carry dead weight at the sleeve. Porter governor and Proell governor have heavy dead weight at the sleeve. In porter governor balls are placed at the junction of upper and lower arms. In case of Proell governor the balls are placed at the extension of lower arms. The sensitiveness of watt governor is poor at high speed and this limits its field of application. Porter governor is more sensitive than watt governor. The Proell governor is most sensitive out of these three.

Watt Governor:

This governor was used by James Watt in his steam engine. The spindle is driven by the output shaft of the prime mover. The balls are mounted at the junction of the two arms. The upper arms are connected to the spindle and lower arms are connected to the sleeve.



(a)



(b)

We ignore mass of the sleeve, upper and lower arms for simplicity of analysis. We can ignore the friction also. The ball is subjected to the three forces which are centrifugal force (Fc), weight (mg) and tension by upper arm (T). Taking moment about point O(intersection of arm and spindle axis), we get

$$F_C\ h - mg\ r = 0$$

Since, $F_C = mr\ \omega^2$

$\therefore$ $mr\ \omega^2 h - mg\ r = 0$

or $\omega^2 = \frac{g}{h}$

$$\omega = \frac{2\pi N}{60}$$

$\therefore$ $h = \frac{g \times 3600}{4\pi^2 N^2} = \frac{894.56}{N^2}$

where 'N' is in rpm.

Porter Governor:

A schematic diagram of the porter governor is shown in Figure 5.4(a). There are two sets of arms. The top arms OA and OB connect balls to the hinge O. The hinge may be on the spindle or slightly away. The lower arms support dead weight and connect balls also. All of them rotate with the spindle. We can consider one-half of governor for equilibrium.

Let w be the weight of the ball,

T_1 and T_2 be tension in upper and lower arms, respectively,

F_c be the centrifugal force,

r be the radius of rotation of the ball from axis, and

I is the instantaneous centre of the lower arm.

Taking moment of all forces acting on the ball about I and neglecting friction at the sleeve, we get

$$F_C \times AD - w \times ID - \frac{W}{2} IC = 0$$

or

$$F_C = \frac{wID}{AD} + \frac{W}{2}\left(\frac{ID + DC}{AD}\right)$$

or

$$F_C = w \tan \alpha + \frac{W}{2} (\tan \alpha + \tan \beta)$$

$$F_C = \frac{w}{g} \omega^2 r$$

∴

$$\frac{w}{g} \omega^2 r = w \tan \alpha \left\{1 + \frac{W}{2w}\left(1 + \frac{\tan \beta}{\tan \alpha}\right)\right\}$$

or

$$\omega^2 = \frac{g}{r} \tan \alpha \left\{1 + \frac{W}{2w} (1 + K)\right\} \quad \ldots (5$$

where $$K = \frac{\tan \beta}{\tan \alpha}$$

$\because$ $$\tan \alpha = \frac{r}{h}$$

$\therefore$ $$\omega^2 = \frac{g}{h}\left\{1 + \frac{W}{2w}(1 + K)\right\}$$

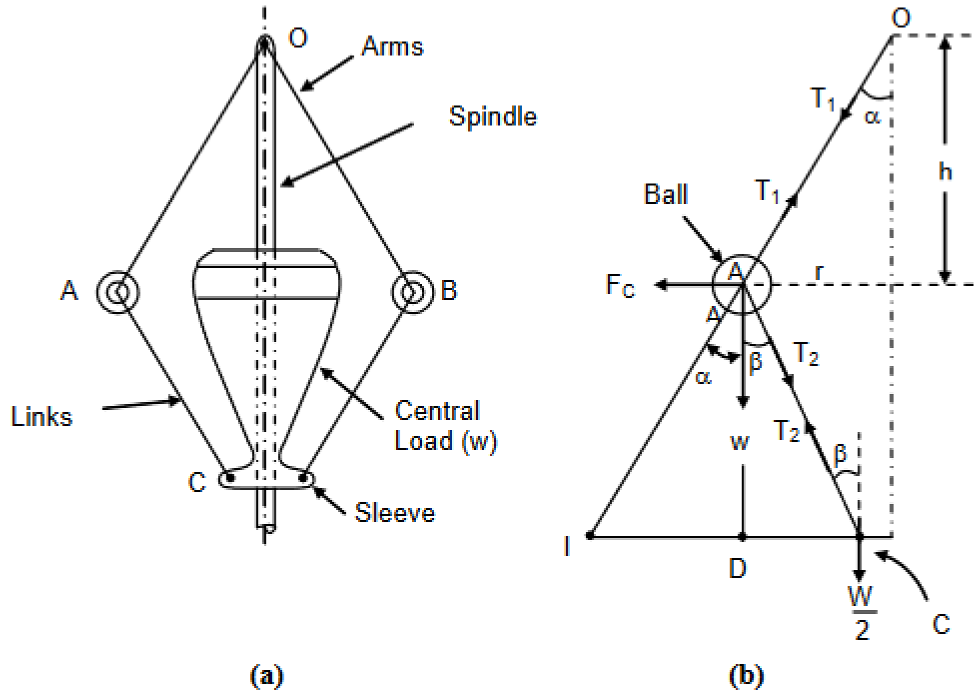


Figure 5.4 : Porter Governor

If friction at the sleeve is f, the force at the sleeve should be replaced by W+ ffor rising and by (W–f) for falling speed as friction apposes the motion of sleeve. Therefore, if the friction at the sleeve is to be considered,Wshould be replaced by (W±f).

$$\omega^2 = \frac{g}{h}\left\{1 + \frac{(W \pm f)}{2w}(1 + K)\right\}$$

Spring controlled centrifugal governor:In these governors springs are used to counteract the centrifugal force. They can be designed to operate at high speeds. They are comparatively smaller in size. Their speed range can be changed by changing the initial setting of the spring. They can work with inclined axis of rotation also. These governors may be very suitable for IC engines, etc.

The most commonly used spring controlled centrifugal governors are :(a)Hartnell governor

(b)Wilson-Hartnell governor

(c)Hartung governor

Hartnell Governor: The Hartnell governor is shown in Figure 5.5. The two bell crank levers have been provided which can have rotating motion about fulcrums Oand O′. One end of each bell crank lever carries a ball and a roller at the end of other arm. The rollers make contact with the sleeve. The frame is connected to the spindle. A helical spring is mounted around the spindle between frame and sleeve. With the rotation of the spindle, all these parts rotate. With the increase of speed, the radius of rotation of the balls increases and the rollers lift the sleeve against the spring force. With the decrease in speed, the sleeve moves downwards. The movement of the sleeve is transferred to the throttle of the engine through linkage.

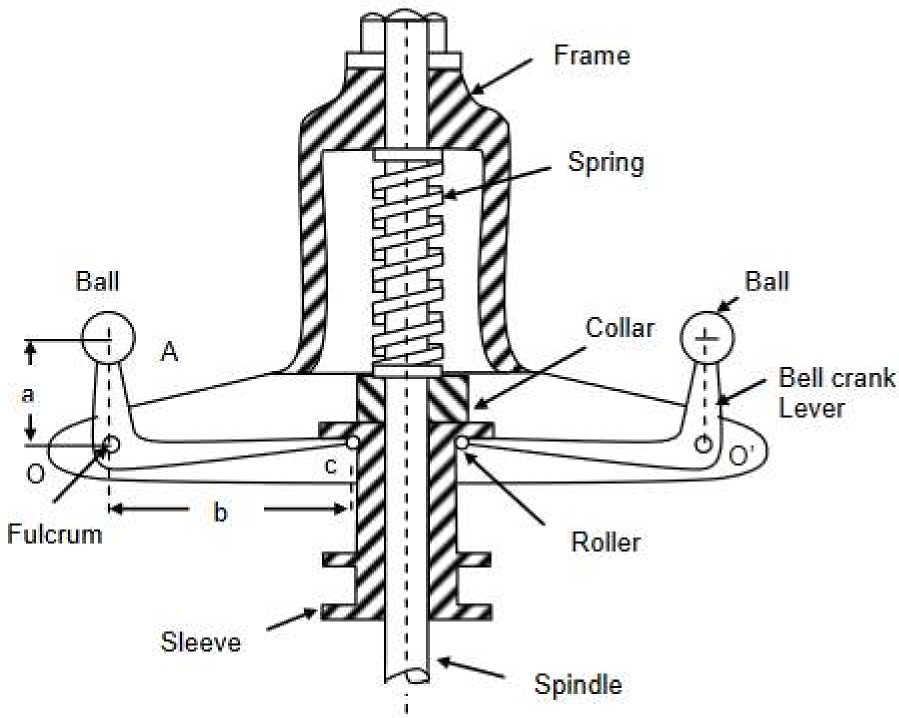


Figure 5.5 : Hartnell Governor

Characteristics of Governors:

Different governors can be compared on the basis of following characteristics.

- Stability: A governor is said to be stable when for every speed within the working range there is a definite configuration i.e. there is only one radius of rotation of the governor balls at which the governor is in equilibrium. For a stable governor, if the equilibrium speed increases, the radius of governor balls must also increase.
- Sensitiveness of Governors: If a governor operates between the speed limits N1 and N2, then sensitiveness is defined as the ratio of the mean speed to the difference between the maximum and minimum speeds. Thus,N1 = Minimum equilibrium speed, N2 = Maximum equilibrium speed, and

N = Mean equilibrium speed $= \frac{N_1+N_2}{2}$

Sensitiveness of the governor

$$= \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

$$= \frac{2(\omega_2 - \omega_1)}{\omega_1 + \omega_2}$$

- Isochronous Governors: A governor is said to be isochronous when the equilibrium speed is constant (i.e. range of speed is zero) for all radii of rotation of the balls within the working range, neglecting friction. The isochronisms are the stage of infinite sensitivity. The isochronous governor is not of practical use because the sleeve will move to one of its extreme positions immediately the speed deviates from the isochronous speed.
- Hunting : Hunting is the name given to a condition in which the speed of the engine controlled by the governor fluctuates continuously above and below the mean speed. It is caused by a governor which is too sensitive and which, therefore, changes by large amount the supply of fuel to the engine.

Flywheel:

A flywheel used in machines serves as a reservoir which stores energy during the period when the supply of energy is more than the requirement and releases it during the period when the requirement of energy is more than supply.

For example, in I.C. engines, the energy is developed only during power stroke which is much more than the engine load, and no energy is being developed during suction, compression and exhaust strokes in case of four stroke engines and during compression in case of two stroke engines. The excess energy developed during power stroke is absorbed by the flywheel and releases it to the crankshaft during other strokes in which no energy is developed, thus Flywheel rotating the crankshaft at a uniform speed. when the flywheel absorbs energy, its speed increases and when it releases, the speed decreases. Hence a flywheel does not maintain a constant speed; it simply reduces the fluctuation of speed.

In other words, a flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation. It does not control the speed variations caused by the varying load.

Coefficient of fluctuation of speed:

The difference between the maximum and minimum speeds during a cycle is called the ***maximum fluctuation of speed***. The ratio of the maximum fluctuation of speed to the mean speed is called ***coefficient of fluctuation of speed***.

Let N_1 = Maximum speed in r.p.m. during the cycle,

N_2 = Minimum speed in r.p.m. during the cycle, and

N = Mean speed in r.p.m. = $\frac{N_1 + N_2}{2}$

∴ Coefficient of fluctuation of speed,

$$C_S = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \quad \text{...(In terms of angular speeds)}$$

$$= \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2} \quad \text{...(In terms of linear speeds)}$$

Maximum Fluctuation of Energy: A turning moment diagram for a multi-cylinder engine is shown by a wavy curve in Fig. 22.4.

The horizontal line AG represents the mean torque line. Let a1, a3, a5 be the areas above the mean torque line and a2, a4 and a6 be the areas below the mean torque line. These areas represent some quantity of energy which is either added or subtracted from the energy of the moving parts of the engine.

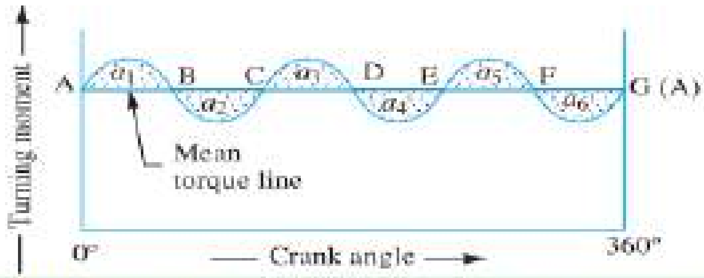


Fig. 22.4. Turning moment diagram for a multi-cylinder engine.

Let the energy in the flywheel at $A = E$, then from Fig. 22.4, we have

Energy at $B = E + a_1$

Energy at $C = E + a_1 - a_2$

Energy at $D = E + a_1 - a_2 + a_3$

Energy at $E = E + a_1 - a_2 + a_3 - a_4$

Energy at $F = E + a_1 - a_2 + a_3 - a_4 + a_5$

Energy at $G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$ = Energy at A

Let us now suppose that the maximum of these energies is at B and minimum at E.

$\therefore$ Maximum energy in the flywheel

$$= E + a_1$$

and minimum energy in the flywheel

$$= E + a_1 - a_2 + a_3 - a_4$$

$\therefore$ Maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$

$$= (E + a_1) - (E + a_1 - a_2 + a_3 - a_4) = a_2 - a_3 + a_4$$

Coefficient of fluctuation of energy:

It is defined as the ratio of the maximum fluctuation of energy to the work done per cycle. It is usually denoted by C_E. Mathematically, coefficient of fluctuation of energy,

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

The workdone per cycle may be obtained by using the following relations:

1. Workdone / cycle $= T_{mean} \times \theta$

where T_{mean} = Mean torque, and

θ = Angle turned in radians per revolution

= 2π, in case of steam engines and two stroke internal combustion engines.

= 4π, in case of four stroke internal combustion engines.

The mean torque (T_{mean}) in N-m may be obtained by using the following relation *i.e.*

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega}$$

where P = Power transmitted in watts,

N = Speed in r.p.m., and

ω = Angular speed in rad/s = $2\pi N / 60$

2. The workdone per cycle may also be obtained by using the following relation:

$$\text{Workdone / cycle} = \frac{P \times 60}{n}$$

where n = Number of working strokes per minute.

= N, in case of steam engines and two stroke internal combustion engines.

= $N/2$, in case of four stroke internal combustion engines.

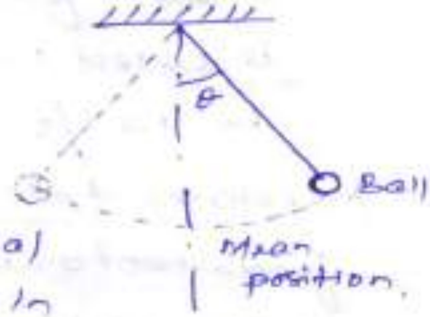
Difference between Flywheel and Governor:

Flywheel	Governor
Flywheel reduces the fluctuation of speed during the thermodynamic cycles, but it does not maintain a constant speed.	Governor is a device to control the speed variation caused by the varying load.
The working of a flywheel does not depend upon the change in load or output required.	Governor operation depends upon the variation of load.
The operation of flywheels is continuous from cycle to cycle.	The operation of a governor is intermittent.
Speed control in a single cycle	Speed control over a period of time
The function of a flywheel is to store energy when mechanical energy is more than required for the operation and release the same when the available energy is less than required. Its inertia helps to run machines at a dead center.	The function of a governor is to regulate the fuel supply according to the load requirement and run the machine at a constant speed irrespective of the output required.
Do not have any control over the supply of charge or fuel.	Control the supply of fuel to the engine
It is relatively heavy and has large inertia force.	It's a light machine part
It is used in engines and fabricating machines such as punching machines, rolling mill, etc.	Governors are provided on engines and turbines.
It is desired where the fluctuation in input torque. e.g.: four stroke engine	Desired where the constant speed required e.g. Generator (there is even electronic governor for diesel generator)

— 'VIBRATION' —

Basic concept:-

The mass is inherent of body and elasticity causes relative motion among it's parts. When the body particles are displaced by the application of external force, the internal forces in the form of elastic energy are present in the body. These forces try to bring the body to it's original position. At equilibrium position, the whole of the elastic energy is converted into kinetic energy and body continues to move in opposite direction because of it. The whole of the kinetic energy is again converted into elastic energy due to which the body again returns to the equilibrium position. This way vibratory motion is repeated with exchange of energy. This phenomenon is called vibration.



— swinging of simple pendulum shown in the fig. is an example of vibration.

Definitions:-

Periodic motion → a motion repeating itself after equal interval of time.

Time period → time taken to complete one cycle.

frequency → No of cycles per unit time.

Amplitude → max^m displacement of vibrating body from its equilibrium position

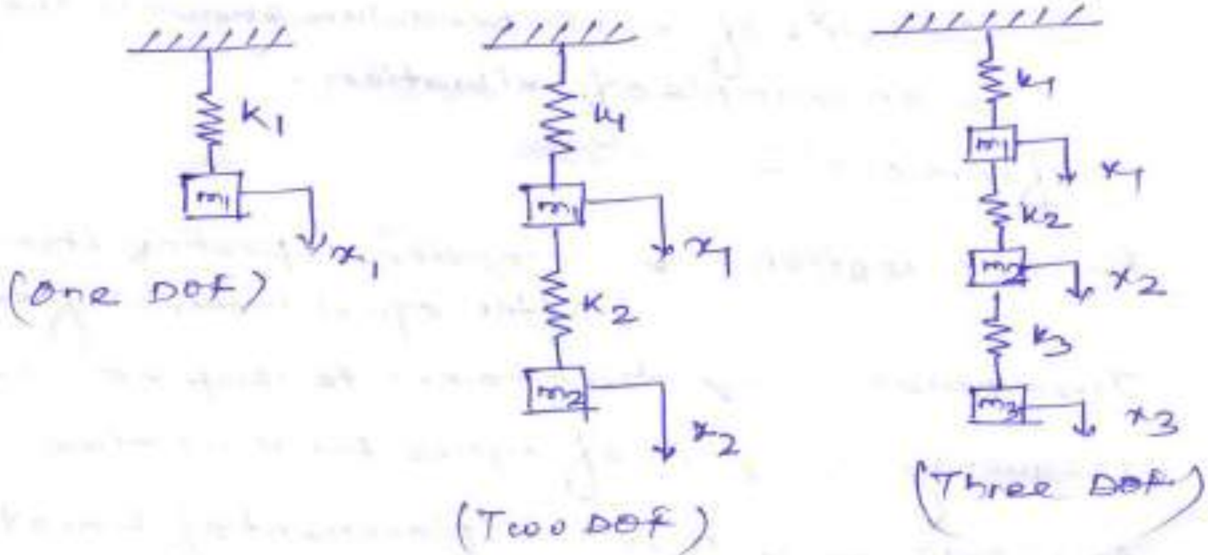
Natural frequency:- When no external force acts on the system after giving it an initial displacement the body vibrates. These vibrations are called free vibrations and their frequency is called natural frequency. It is expressed in rad/s or Hertz.

Fundamental mode of vibration:- The fundamental mode of vibration of a system is the mode having the lowest natural frequency.

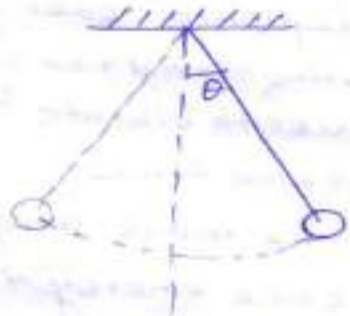
Degree of freedom:- The min^m no of independent coordinates required to specify the motion of a system at any instant is known as degrees of freedom of the system.

- In general it is equal to the no of independent displacements that are possible. This number varies from zero to infinity.

Example of one, two and three degree of freedom system are shown in the figures.



Simple Harmonic Motion:- The motion of a body to and fro about a fixed point is called Simple Harmonic motion. The motion is periodic and it's acceleration is always directed towards the mean position and is proportional to it's distance from mean position. The motion of a simple pendulum is an example of SHM.



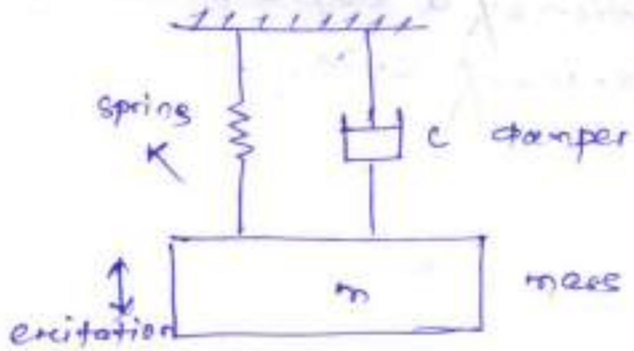
Damping:- It's the resistance to the motion of a vibrating body. The vibrations associated with this resistance are known as damped vibrations.

Resonance:- When the frequency of external excitation is equal to the natural frequency of a vibrating body, the amplitude of vibration becomes excessively large. This phenomenon is called resonance.

× **Parts of a Vibrating system:-**

A simple vibratory system consists of three elements namely mass, the spring and damper. In a vibratory body there is exchange of energy from one form to another. Energy is stored by mass in the form of KE $(\frac{1}{2} m \dot{x}^2)$, in the spring in the form of PE $(\frac{1}{2} K x^2)$ and dissipated in the damper in the form of heat energy which opposes the motion of the system.

Energy enters the system with the application of external force, known as excitation. The excitation disturbs the mass from its mean position and it goes up and down from its mean position.



The kinetic energy is converted to potential energy and vice versa. This sequence goes on repeating and the system continues to vibrate. At the same time damping force $c\dot{x}$ acts on the mass and opposes its motion. Thus some energy is dissipated in each cycle of vibration due to damping. After some time free vibration die out and the system remains at its static equilibrium position. A basic vibratory system is shown in figure.

The equation of motion for such a vibratory system is

$$m\ddot{x} + c\dot{x} + Kx = 0$$

where $c\dot{x}$ = damping force

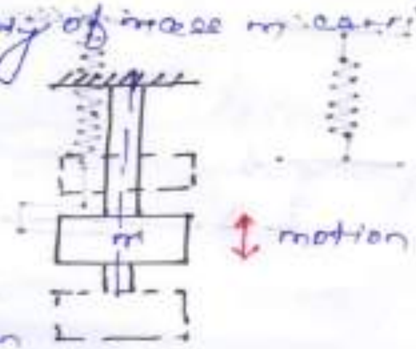
Kx = spring force

$m\ddot{x}$ = inertia force.

Types of vibrations :-

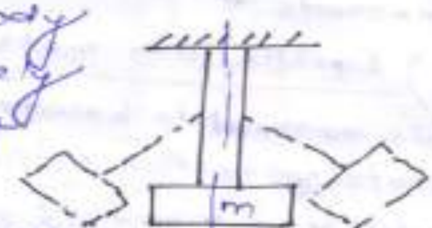
(i) Longitudinal vibrations :-

considering a case where a body of mass m carried on one end of a weightless spindle, the other end being fixed. If the mass 'm' moves up and down parallel to the spindle axis, it is said to execute longitudinal vibration.



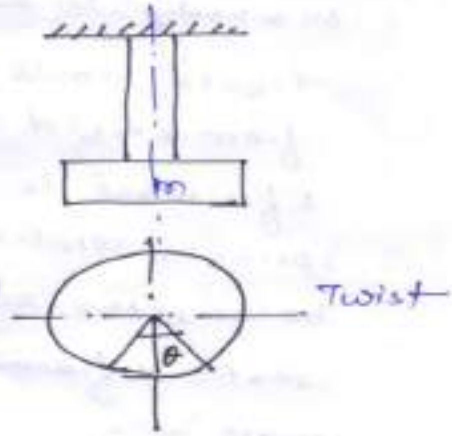
(ii) Transverse vibrations :-

When the particles of the body or shaft move approximately ⊥r to the axis of the shaft as shown in the figure, the vibration caused is known as transverse vibration.



(iii) Torsional Vibrations :-

If the spindle get alternately twisted and untwisted on account of vibratory motion of the suspended disc, then it is undergoing torsional vibration



Transient vibrations :-

In ideal systems the free vibration continues indefinitely as there is no damping. The amplitude of vibration decays continuously because of damping (In practical system) and vanishes ultimately. Such vibration is real system is called transient vibration.

Causes of Vibration:

(1) Unbalance force in the different parts of the machine.

(2) Lack of lubricants between two mating surface.

(3) External load or force which makes system vibrant.

(4) Earthquakes

(5) Lack of balancing of force in machine part.

(6) Worn out or defective parts of the machine

(7) Improper meshing of gear ~~the teeths~~

(8) Friction between the moving part & stationary part.

(9) looseness of parts (loose bolts, excessive clearance)

Effects of Vibration:

(1) Severe machine damage
(2) High power consumption.
(3) Unnecessary maintenance
(4) Machine unavailability due to breakdow
(5) Occupational hazards
(6) Produces excessive stresses
(7) Reduces machine element life.
(8) Produces undesirable noise

Remedies of Vibration:

(1) Using shock absorber in vehicles
(2) Using isolator between moving parts and stationary parts
(3) Reducing friction between two parts
(4) Lubricating the mating surface of two parts
(5) Reducing the pressure and speed.
(6) Reducing the unbalance force on the machine parts.
(7) Replacing the defective parts
(8) Performing Periodic maintenance.

BALANCING:

Introduction:-

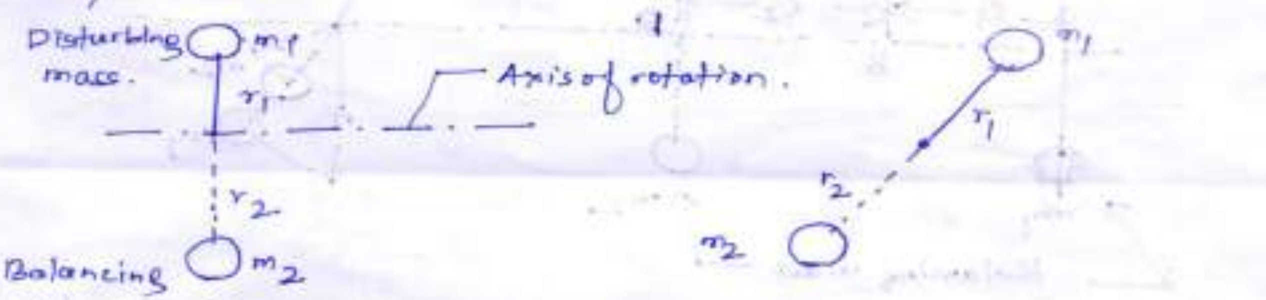
Machines have several rotating parts. Some of them have reciprocating motion e.g. piston and some of them have rotating motion e.g. crankshaft. If these moving parts are not in complete balance, inertia force generation would lead to vibration, noise, wear and tear of the parts.

– Balancing plays a major role in designing these systems to reduce unbalance to an acceptable limit.

Balancing of Single Revolving mass:-

(i) Balancing in same plane (ii) balancing in different plane.

(i) **Balancing and disturbing mass revolve in same plane:-**



Let m_1 = mass attached to the shaft

ω = angular velocity of the mass in rad/s.

r_1 = distance of c.g. of the mass from axis of rotation.

In order to counteract the disturbing force e.g. the centrifugal force due to m_1, a counter mass m_2 at a radius r_2 is placed in the same plane, such that the centrifugal forces due to the two masses are equal and opposite.

Mathematically, $F_{c_1} = m_1 \omega^2 r_1$

balancing force $F_{c_2} = m_2 \omega^2 r_2$

For balancing, $F_{c_1} = F_{c_2}$

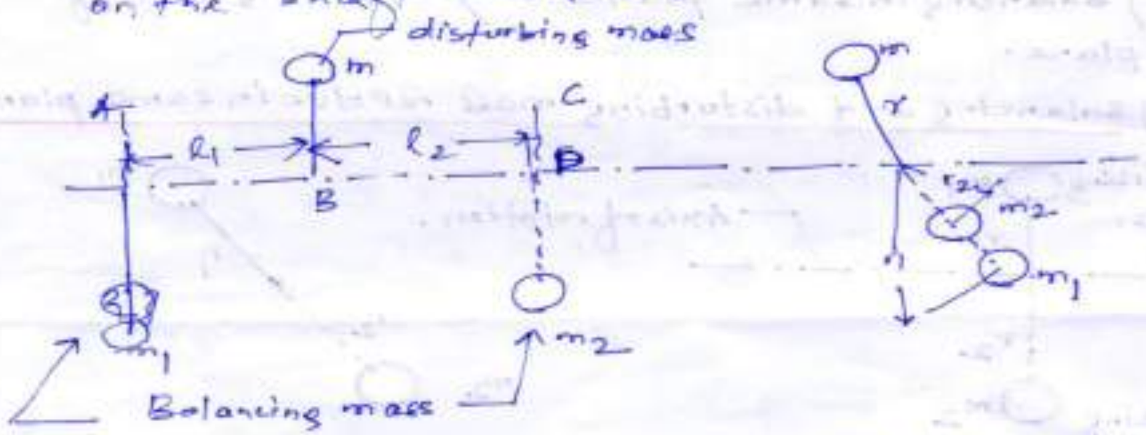
$\Rightarrow m_1 \omega^2 r_1 = m_2 \omega^2 r_2$

$$\Rightarrow \boxed{m_1 r_1 = m_2 r_2}$$

Generally the value of r_2 is kept is larger to reduce the value of balancing mass m_2

(ii) Balancing and Disturbing mass revolve in different plane! —

In case the balancing and the disturbing mass lie in different planes, the disturbing mass can not be balanced by a single mass as there will be a couple left unbalanced. In such case at least two balancing masses are required for complete balancing. The three masses are arranged in such a way that the resultant force and couple on the shaft are zero.



Let m = mass of disturbing body acting in plane B
m_1 = mass of balancing weight acting in plane A
m_2 = mass of balancing weight acting in plane B
l_1 = distance betn plane A and B
l_2 = distance betn plane B and C
$l = l_1 + l_2$
r, r_1, r_2 → distances of CG of m, m_1, m_2 respectively

Now $F_c = m\omega^2 r$

$F_{c1} = m_1\omega^2 r_1$

$F_{c2} = m_2\omega^2 r_2$

For balancing the centrifugal force of disturbing mass must be equal to the sum of centrifugal force of balancing mass

$F_c = F_{c1} + F_{c2}$

or $m\omega^2 r = m_1\omega^2 r_1 + m_2\omega^2 r_2$

$\Rightarrow \boxed{mr = m_1 r_1 + m_2 r_2}$

For complete balance, sum of moments should be zero.

Taking moment about D

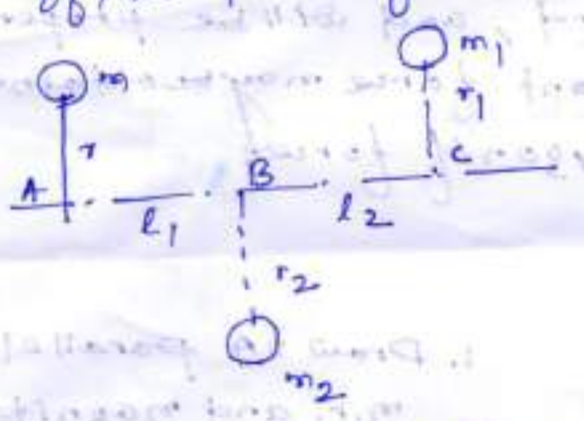
$(l_1+l_2)\, F_{c_1} = F_c \cdot l_2$

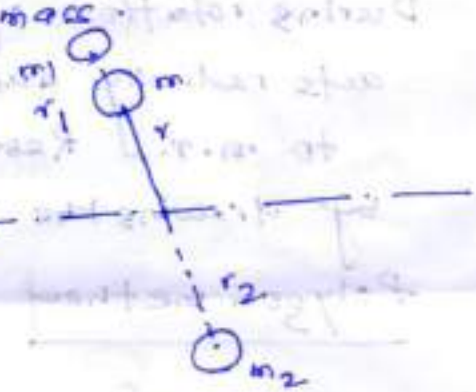
$\Rightarrow l\, m_1\omega^2 r_1 = m\omega^2 r \cdot l_2$

$\Rightarrow \boxed{m_1 r_1 = \dfrac{m r l_2}{l}}$

where $\boxed{l = l_1 + l_2}$

Case-II → Plane of disturbing mass lies on one side of the plane of balancing mass.





We have ~~$F_c = F_{c_1} + F_{c_2}$~~

$F_c + F_{c_1} = F_{c_2}$

or $m\omega^2 r + m_1\omega^2 r_1 = m_2\omega^2 r_2$

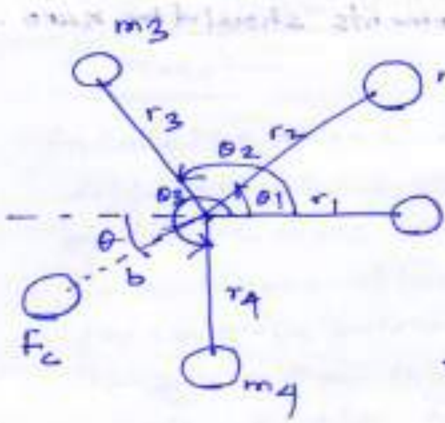
or $mr + m_1 r_1 = m_2 r_2$

couple equation can be written by taking moment at B

$F_c \cdot l_1 = F_{c_1} \cdot l_2$

or $\boxed{m r l_1 = m_1 r_1 l_2}$

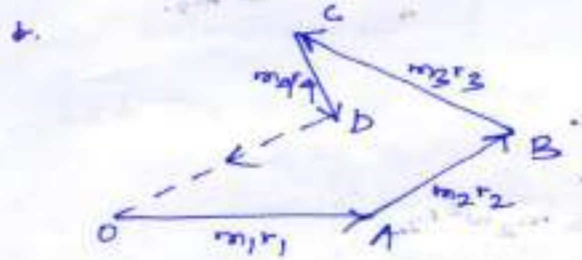
Balancing of several masses revolving in same plane :-



Consider any no of masses say four m_1, m_2, m_3 and m_4 rigidly attached to the shaft and lie in same plane. Let r_1, r_2, r_3 and r_4 be the radii of rotation of masses. Their relative positions are indicated by angles $\theta_1, \theta_2, \theta_3$ and θ_4.

During rotation of shaft, centrifugal force acts radially outward, and will be proportional to m.r. Resultant force may be obtained by drawing the polygon of forces

Polygon method



1. Draw OA parallel to m_1r_1 and magnitude $OA = m_1r_1$
2. From A draw AB parallel and equal to m_2r_2.
3. From B draw BC parallel and equal to m_3r_3
4. from C draw CD parallel and equal to m_4r_4
5. Join D with O, DO represents the direction and magnitude of balanced force.

Analytical method

Resolving each force horizontally and vertically
Resultant vertical component is

$$F_V = m_1r_1 \sin\theta + m_2r_2 \sin\theta_1 + m_3r_3 \sin\theta_2$$

$F_H = m_1 r_1 \cos 0 + m_2 r_2 \cos\theta_1 + m_3 r_3 \cos\theta_2 + m_4 r_4 \cos\theta_3$ —— (2)

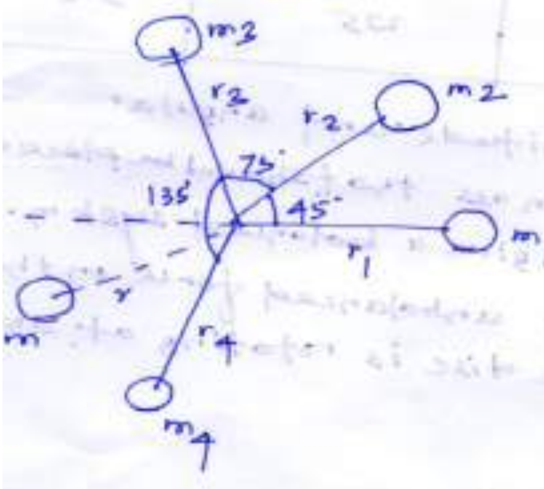
The resultant B.b may be written as

$$\text{B.b} = \sqrt{F_V^2 + F_H^2}$$

And it's direction $\tan\theta = \frac{F_V}{F_H}$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{F_V}{F_H}\right)$$

Q.1 Four masses m_1, m_2, m_3 and m_4 having their radii of rotations as 200 mm, 150 mm, 250 mm and 300 mm are 200 kg, 300 kg, 240 kg and 260 kg. The angle betn the successive masses are 45°, 75° and 135°. Find position and magnitude of the balance mass required if it's radius of rotation is 200 mm.



We have

$m_1 = 200\,kg$ $m_2 = 300\,kg$

$m_3 = 240\,kg$ $m_4 = 260\,kg$.

$\theta_1 = 0$ $\theta_2 = 45°$

$\theta_3 = 45 + 75 = 120°$

$\theta_4 = 120° + 135 = 255°$

$r_1 = .2\,m$ $r_2 = 0.15\,m$ $r_3 = .25\,m$

$r_4 = 0.3\,m$.

$r = 0.2\,m$

Analytical method

$\Sigma F_V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 + m_4 r_4 \sin\theta_4$

$= 40 \sin 0° + 45 \sin 45° + 60 \sin 120° + 78 \sin 255°$

$= 8.439\ kg\text{-}m$.

$\Sigma F_H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + m_4 r_2 \cos\theta_4$

$= 40 \cos 0 + 45 \cos 45° + 60 \cos 120 + 78 \cos 255° = 21.62\ kg\text{m}$.

Resultant force

$F = \sqrt{F_V^2 + F_H^2} = 23.2$ Kg-m.

Now $m \cdot r = 23.2$ Kg m

$\Rightarrow m = \dfrac{23.2}{0.2} = \boxed{116 \text{ kg.}}$

Direction

$\tan\theta = \dfrac{\Sigma F_V}{\Sigma F_H}$

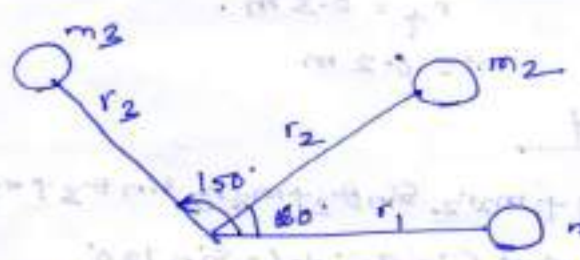
$\Rightarrow \theta = \tan^{-1}\left(\dfrac{8.43}{21.68}\right) = \boxed{~~201.5^\circ~~}$ 21.29°

Direction from m_1 = 180 + 21.29 = 201.5°

Q.2 A circular disc rotating around a vertical spindle, has the following masses placed on it.

mass	θ, wrt X-X	Distance from centre (mm)	Magnitude
m_1	0	260	2.5
m_2	60	300	3.5
m_3	150°	225	5.0

Determine the magnitude and angular position of a mass that should be placed at 262.5 mm to give a balanced system. Also determine the unbalanced force on the spindle when the disc is rotating at 950 rpm.



$\Sigma F_V = m_1 r_1 \sin 0 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3$

$= 0.65 \sin 0 + 1.05 \sin 60^\circ + 1.125 \sin 150^\circ$

$= 1.471$ Kg m.

Resultant force $F = 1.484$ kgm.

So $m \cdot r = 1.484$ kgm

$\Rightarrow \boxed{m = 5.653 \text{ kg}}$

$\theta = \tan^{-1}\left(\frac{1.471}{0.2007}\right) = \boxed{82.23^\circ}$

Direction from $m_1 = \boxed{262.23^\circ}$

Magnitude of Resultant force.

$m\omega^2 r = 5.653 \times \left(\frac{2\pi \times 250}{60}\right)^2 \times .2625$

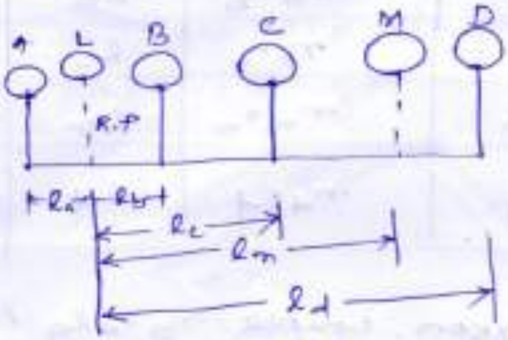
$= \boxed{1017 \text{ N.}}$

Balancing of Several masses Revolving in Different Planes :-

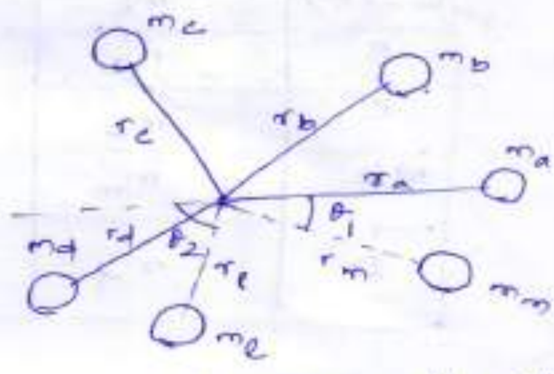
- Balancing of several masses revolving in different planes is done by transfer of the centrifugal force acting in different planes to a single plane, known as reference plane, thereby masses rotating in different planes are transferred to reference plane.

  The effect of transferring the rotating mass m in the reference plane is to generate a centrifugal force, $F_c = m\omega^2 r$ and a couple $C = F_c \cdot l$ in the reference plane where l = distance betn the reference plane and rotating.

- For complete balancing of such system, two conditions must be satisfied.
  1. Resultant centrifugal force must be zero
  2. Resultant couple must be zero.



(position of planes)



(Angular position of masses)

Let's consider several masses m_a, m_b, m_c and m_d revolving in planes A, B, C and D respectively.

Two masses for balancing are used because of the following use!

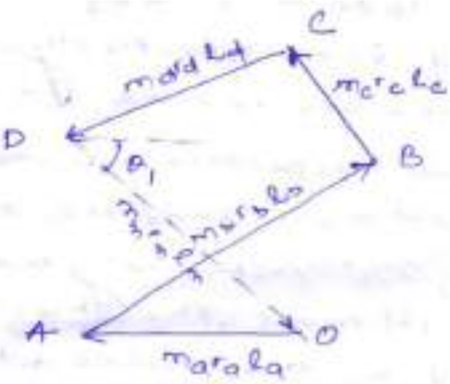
1. If a single mass is used the system will be difficult to handle.
2. If more than two masses are used, no of unknown parameters will be more than no of equations.

Procedure:-

1. Take one plane L as the reference plane. Distance to the left of this plane are taken with minus sign and those to right with +ve sign.
2. Tabulate the forces and couples as shown in the table.

Plane (1)	Mass (m) (2)	radius (r) (3)	centrifugal force ÷ ω^2 (mr) (4)	Distance from R.P. (l) (5)	couple ÷ ω^2 (mrl) (6)
A	m_a	r_a	$m_a r_a$	$-l_a$	$-m_a r_a l_a$
L (RP)	m_l	r_l	$m_l r_l$	0	0
B	m_b	r_b	$m_b r_b$	l_b	$m_b r_b l_b$
C	m_c	r_c	$m_c r_c$	l_c	$m_c r_c l_c$
M	m_m	r_m	$m_m r_m$	l_m	$m_m r_m l_m$
D	m_d	r_d	$m_d r_d$	l_d	$m_d r_d l_d$

3. Draw the couple polygon. Couple $m_a r_a l_a$ is -ve wrt RP. so the couple $(-m_a r_a l_a)$ is drawn radially inwards as it is in reverse direction of Om_a. couple $m_b r_b l_b$ is +ve wrt RP so it is drawn in the direction of Om_b. Similarly couples $m_c r_c l_c$ and $m_d r_d l_d$ are drawn in the directions of Om_c and Om_d respectively.



(couple polygon)

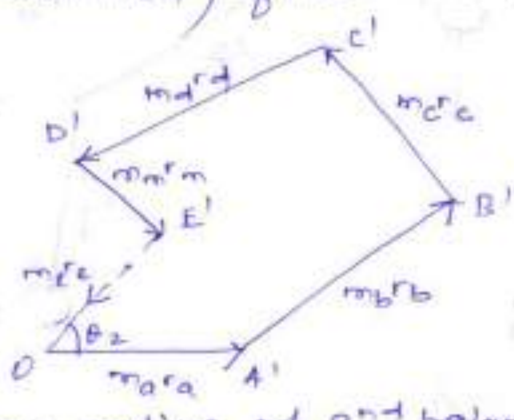
couple $m_m r_m l_m$ is the closing side. The balancing couple OD is proportional to $m_m r_m l_m$.

If the balancing radius r_m is known, balancing mass m_m can be obtained in magnitude and direction.

$$\boxed{m_m = \frac{OD}{r_m \cdot l_m}} \qquad \boxed{OD = m_m r_m l_m}$$

Thus m_m in plane M can be determined and angle θ_1 can be measured.

4. We can find other balancing mass m_l in plane L with the help of force polygon tabulated in column (4) of the table.



If the radius of 2nd balancing mass m_l is known m_l can be found in plane L and its angle of inclination θ_2 with horizontal may be measured.

Q2 A rotating shaft carries four unbalanced masses 18 kg, 14 kg, 16 kg and 12 kg at radii 5cm, 6cm, 7cm and 6cm respectively. The 2nd, 3rd and 4th masses revolve in planes 8cm, 16cm and 28cm respectively measured from the plane of 1st mass and are angularly located at 60°, 135°, 270° respectively measured anticlockwise from 1st mass. The shaft is dynamically balanced by two masses both located at 5cm radii and revolving in planes midway between those of 1st and 2nd masses and 3rd and 4th masses. Determine graphically the magnitude of the masses and their respective angular positions.

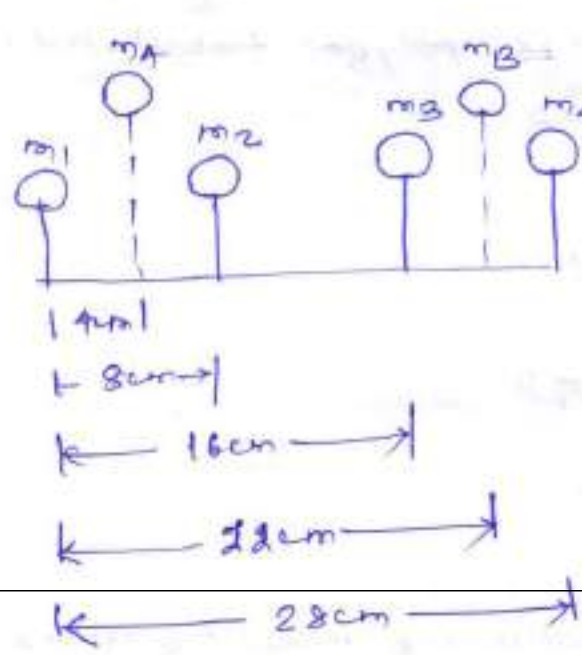
Given data :- $m_1 = 18$ kg $m_2 = 14$ kg $m_3 = 16$ kg

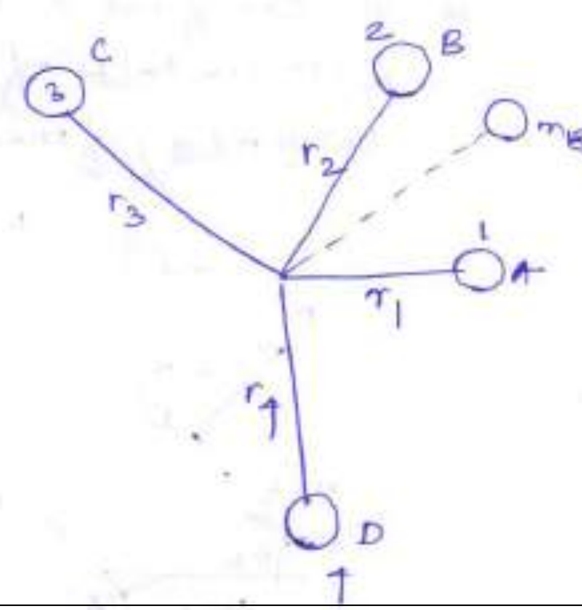
$m_4 = 12$ kg.

$r_1 = 5$cm $r_2 = 6$cm $r_3 = 7$cm $r_4 = 6$cm.

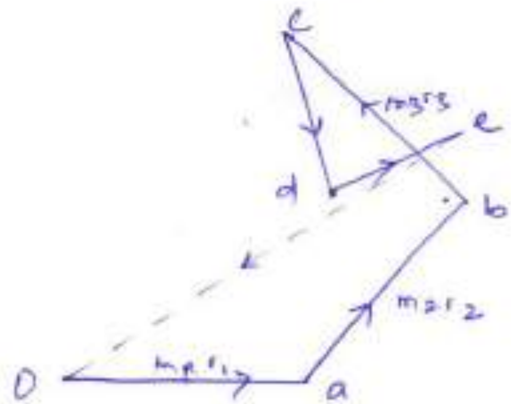
$\theta_1 = 0$ $\theta_2 = 60°$ $\theta_3 = 135°$ $\theta_4 = 270°$

Let the two balancing masses are m_A and m_B

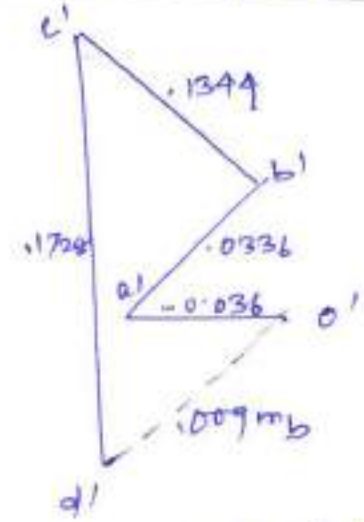




plane	mass (m) Kg	Radius (r) m	centrifugal force ÷ w^2 (mr)	Distance from RP (l)	couple ÷ w^2 (mrl)
1	18	0.05	0.9	-.04	-0.036
A	m_A	0.05	$0.05m_A$	0	0
2	14	.06	0.84	.04	0.0336
3	16	.07	1.12	.12	0.1344
B	m_B	.05	$0.05m_B$	.18	$.009m_B$
4	12	.06	0.72	.24	0.1728



(force polygon)



$m_A = \frac{1.575}{0.05} = 31.5$ Kg.

$\theta_A = 220^\circ$ (from plane 1)

(2nd)

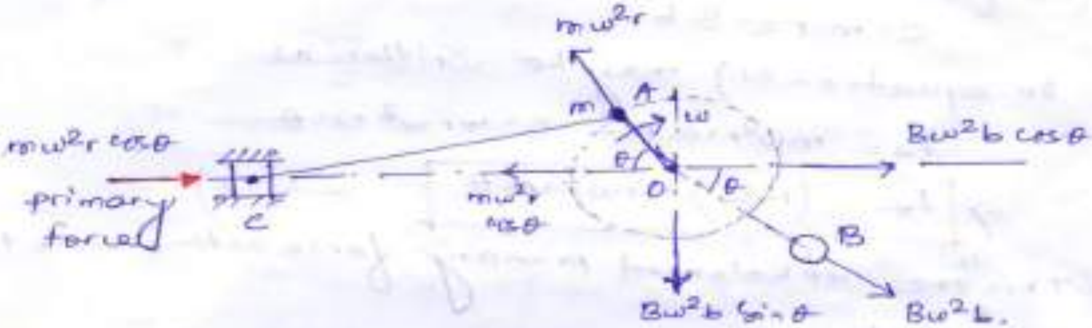
$m_B = \frac{o'd'}{.009}$

$= \frac{.12}{.009} = 13.33$ Kg.

$\theta_B = 25^\circ$

(1st)

Partial Primary Balancing :-



Consider a slider crank mechanism OAC. A primary unbalanced force $m\omega^2 r\cos\theta$ is required to accelerate the reciprocating mass, which acts along the direction from O to C. So balancing of primary force is considered equivalent to the component and parallel to the line of stroke, of the centrifugal force produced by an equal mass 'm' attached to the crank and and rotating at 'r' radius. To balance this force a rotating counter mass B is placed at a radius b, directly opposite to crank.

For complete balancing

$$B\omega^2 b\cos\theta = m\omega^2 r\cos\theta$$

$$\Rightarrow \boxed{B\cdot b = m\cdot r}$$

- However the vertical component of rotating mass B, of magnitude $B\omega^2 b\sin\theta$ remains unbalanced.

Now the resultant disturbing force parallel to the line of stroke is

$$F_H = m\omega^2 r\cos\theta - B\omega^2 b\cos\theta$$

$$\Rightarrow \boxed{F_H = (m r - B\cdot b)\,\omega^2\cos\theta} \quad \text{—— (1)}$$

If $m\cdot r = B\cdot b$, the primary disturbing force is zero and the system will be unbalanced because of the vertical component of force.

Practically, a compromise is made and only a fraction c of reciprocating mass is balanced i.e,

$$c \cdot m \cdot r = B \cdot b.$$

so equation (1) may be written as

$$F_H = m\omega^2 r\cos\theta - c \cdot mr\omega^2 \cos\theta$$

$$\Rightarrow \boxed{F_H = (1-c)\, mr\omega^2 \cos\theta} \quad \text{--- (2)}$$

This is the unbalanced primary force acting along the line of stroke.

The unbalanced force ⊥r to the line of stroke is

$$F_V = B\omega^2 b \sin\theta = c m\omega^2 r \sin\theta \quad \text{--- (3)}$$

so the resultant unbalanced force

$$F = \sqrt{F_H^2 + F_V^2}$$

$$\Rightarrow \boxed{F = m\omega^2 r\sqrt{(1-c)^2\cos^2\theta + c^2\sin^2\theta}} \quad \text{--- (4)}$$

The value of c is kept betn $1/2$ to $3/4$.

The value of unbalanced force is minm when

$$c = \frac{1}{2}$$

$$F_{min} = m\omega^2 r\sqrt{\left(\tfrac{1}{2}\right)^2\cos^2\theta + \left(\tfrac{1}{2}\right)^2\sin^2\theta}$$

$$\Rightarrow \boxed{F_{min} = \frac{m\omega^2 r}{2}}$$

Q1 The following data relate to a single-cylinder reciprocating engine:

mass of reciprocating parts = 40 kg.

mass of revolving part = 30 kg at 180 mm radius.

speed = 150 rpm

turned 45° from the TDC.

We have $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 150}{60} = 15.7 \text{ rad/s}$.

$r = \frac{350}{2} = 175 \text{ mm}$.

(i) mass to be balanced $= c.m + m_p$

where m_p = mass of crankpin

m = reciprocating mass

c = fraction of reciprocating mass

So total mass to be balanced $= 0.6 \times 40 + 30$

$= 54 \text{ kg}$.

Now $B.b = m.r$

$B \times 320 = 54 \times 180$

Complete Balancing of Reciprocating Parts of an engine:-

For complete balancing of reciprocating parts of an engine, the following conditions must be satisfied:

- Primary force polygon must close
- Primary couple polygon must close
- secondary force polygon must close
- Secondary couple polygon must close.

Power Transmission

Power transmission devices are very commonly used to transmit power from one shaft to another. Belts, chains and gears are used for this purpose. When the distance between the shafts is large, belts or ropes are used and for intermediate distance chains can be used. For belt drive distance can be maximum but this should not be more than ten metres for good results. Gear drive is used for short distances.

1. **Belts and ropes** are used when the distance between the axes of the two shafts to be connected is considerable. Such connectors are non-rigid and undergo strain while in motion. These devices are called non-positive drive because of the possibility of slip occurring between the belt and pulley.

2. **Chain drive** is used when the distance between the shaft centers is short and no slip is required. These connectors are referred to as a positive or non-slip drive.

3. **Gears** are used for transmitting motion and power when the distance between the driving & driven shafts is relatively small, and when a constant velocity ratio is desired.

4. **Clutches** are used for power transmission between co-axial shafts.

Belts

In case of belts, friction between the belt and pulley is used to transmit power. In practice, there is always some amount of slip between belt and pulleys, therefore, exact velocity ratio cannot be obtained. That is why, belt drive is not a positive drive. Therefore, the belt drive is used where exact velocity ratio is not required.

The following types of belts shown in Figure 3.1 are most commonly used :

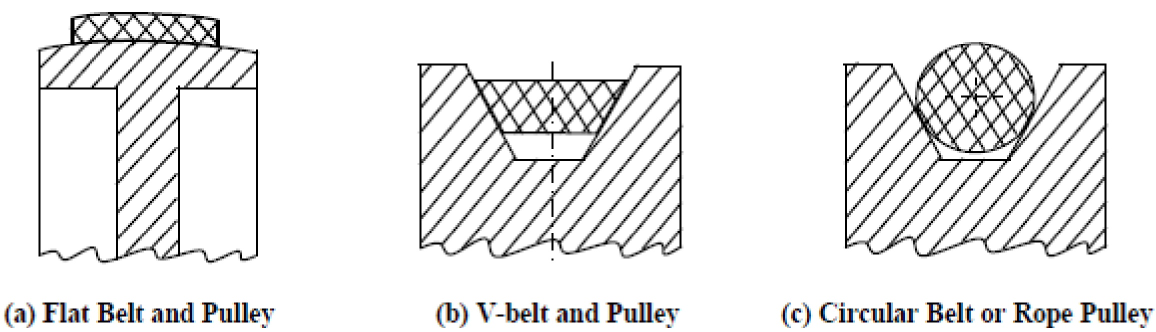


Figure 3.1 : Types of Belt and Pulley

The flat belt is rectangular in cross-section as shown in Figure 3.1(a). The pulley for this belt is slightly crowned to prevent slip of the belt to one side. It utilises the friction between the flat surface of the belt and pulley.

The V-belt is trapezoidal in section as shown in Figure 3.1(b). It utilizes the force of friction between the inclined sides of the belt and pulley. They are preferred when distance is comparative shorter. Several V-belts can also be used together if power transmitted is more.

The circular belt or rope is circular in section as shown in Figure 8.1(c). Several ropes also can be used together to transmit more power.

The belt drives are of the following types :

(a) open belt drive, and

(b) cross belt drive.

Open Belt Drive

Open belt drive is used when sense of rotation of both the pulleys is same. It is desirable to keep the tight side of the belt on the lower side and slack side at the top to increase the angle of contact on the pulleys. This type of drive is shown in Figure 3.2.

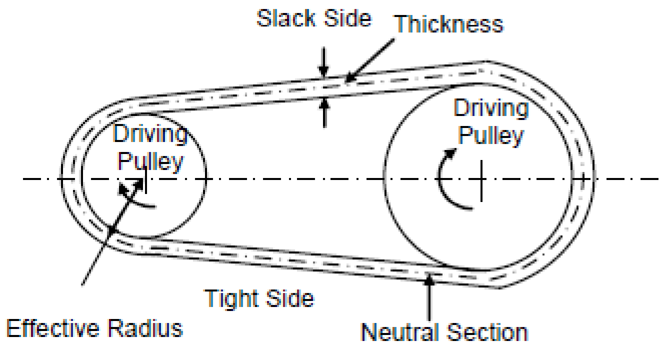


Figure 3.2 : Open Belt Derive

Cross Belt Drive

In case of cross belt drive, the pulleys rotate in the opposite direction. The angle of contact of belt on both the pulleys is equal. This drive is shown in Figure 3.3. As shown in the figure, the belt has to bend in two different planes. As a result of this, belt wears very fast and therefore, this type of drive is not preferred for power transmission. This can be used for transmission of speed at low power.

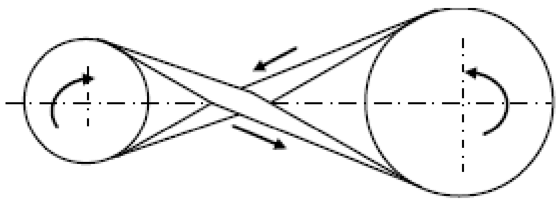


Figure 3.3 : Cross Belt Drive

Since power transmitted by a belt drive is due to the friction, belt drive is subjected to slip and creep.

Let d_1 and d_2 be the diameters of driving and driven pulleys, respectively. N_1 and N_2 be the corresponding speeds of driving and driven pulleys, respectively.

The velocity of the belt passing over the driver

$$V_1 = \frac{\pi d_1 N_1}{60}$$

If there is no slip between the belt and pulley

$$V_1 = V_2 = \frac{\pi d_2 N_2}{60}$$

or, $$\frac{\pi d_1 N_1}{60} = \frac{\pi d_2 N_2}{60}$$

or, $$\frac{N_1}{N_2} = \frac{d_2}{d_1}$$

If thickness of the belt is 't', and it is not negligible in comparison to the diameter,

$$\frac{N_1}{N_2} = \frac{d_2 + t}{d_1 + t}$$

Let there be total percentage slip 'S' in the belt drive which can be taken into account as follows :

$$V_2 = V_1 \left(1 - \frac{S}{100}\right)$$

or $$\frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60} \left(1 - \frac{S}{100}\right)$$

If the thickness of belt is also to be considered

or $$\frac{N_1}{N_2} = \frac{(d_2 + t)}{(d_1 + t)} \times \frac{1}{\left(1 - \frac{S}{100}\right)}$$

or, $$\frac{N_2}{N_1} = \frac{(d_1 + t)}{(d_2 + t)} \times \left(1 - \frac{S}{100}\right)$$

The belt moves from the tight side to the slack side and vice-versa, there is some loss of power because the length of belt continuously extends on tight side and contracts on loose side. Thus, there is relative motion between the belt and pulley due to body slip. This is known as creep.

Length of belt

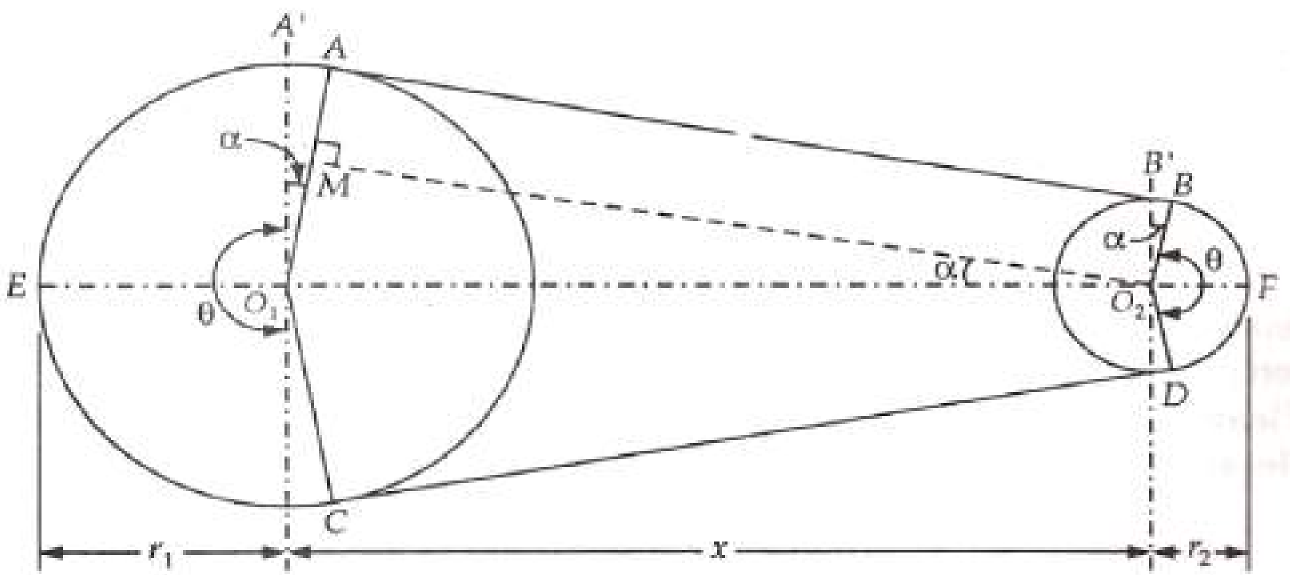
(i) **Open belt system***:* the open belt system in which both the driving an driven pulleys rotate in the same direction.

Let r_1, r_2= radius of the two pulleys

x = distai1ce between $O_1$and O_2; the centers of the two pulleys.

The belt leaves the bigger pulley at *A* and C, and the smaller pulley at Band D.

A line O_2M drawn parallel to *AB* will be perpendicular to O_1A also.



$$\angle A'O_1A = \angle B'O_2B = \angle O_1O_2M = \alpha$$

$$\sin\alpha = \frac{r_1 - r_2}{x}$$

Since α is very small,

$$\sin\alpha = \alpha = \frac{r_1 - r_2}{x}$$

Length of the belt, $l = 2(\text{arc } EA + AB + \text{arc } BF)$

$$\text{arc } EA = r_1 \times \left(\frac{\pi}{2} + \alpha\right) \text{ and arc } BF = r_2 \times \left(\frac{\pi}{2} - \alpha\right)$$

$$AB = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2}$$

$$= \sqrt{x^2 - (r_1 - r_2)^2} = x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2} = x\left[1 - \left(\frac{r_1 - r_2}{x}\right)^2\right]^{1/2}$$

Since $\left(\frac{r_1 - r_2}{x}\right)^2$ is very small, Binomial expansion would give

$$AB = x\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2\right] = x\left[1 - \frac{(r_1 - r_2)^2}{2x^2}\right]$$

$$\therefore \qquad l = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x\left\{1 - \frac{(r_1 - r_2)^2}{2x^2}\right\} + r_2\left(\frac{\pi}{2} - \alpha\right)\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 - r_2) + \left\{x - \frac{(r_1 - r_2)^2}{2x}\right\}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

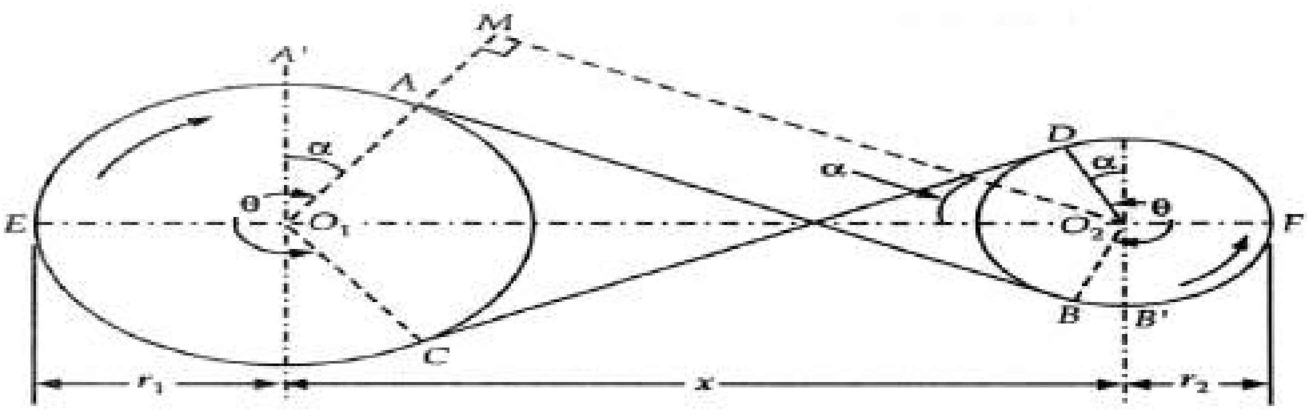
Substituting the value $\alpha = \frac{r_1 - r_2}{x}$, we get

$$l = \pi(r_1 + r_2) + 2\frac{r_1 - r_2}{x} \times (r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{(r_1 - r_2)^2}{x} + 2x$$

(ii) **Crossed belt system**: the crossed belt system in which the driving and the driven pulleys rotate in opposite directions.

The belt leaves the bigger pulley at A and C and the smaller pulley at Band D. A line 0_2M is drawn parallel at AB will be perpendicular to O_1A also.



let r_1 and r_2 = radius of the two pulleys
x = distance between O_1 and O_2; the centres of the two pulleys

angle $A'O_1A$ = angle $B'OB$ = angle O_1O_2M = α

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1A + AM}{O_1O_2} = \frac{r_1 + r_2}{x}$$

Since α is very small,

$$\sin\alpha = \alpha = \frac{r_1 + r_2}{x}$$

length of belt,

$$l = 2(\text{arc } EA + AB + \text{arc } BF)$$

$$\text{arc } EA = r_1\left(\frac{\pi}{2} + \alpha\right) \text{ and } \text{arc } BF = r_2\left(\frac{\pi}{2} + \alpha\right)$$

$$AB = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{x^2 - (r_1 + r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 + r_2}{x}\right)^2} = x\left[1 - \left(\frac{r_1 + r_2}{x}\right)^2\right]^{1/2}$$

Through binomial expansion

$$AB = x\left[1 - \frac{1}{2}\left(\frac{r_1 + r_2}{x}\right)^2\right] = x\left[1 - \frac{(r_1 + r_2)^2}{2x^2}\right]$$

$$l = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x\left\{1 - \frac{(r_1 + r_2)^2}{2x^2}\right\} + r_2\left(\frac{\pi}{2} + \alpha\right)\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 + r_2) + \left\{x - \frac{(r_1 + r_2)^2}{2x}\right\}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$l = \pi(r_1 + r_2) + 2\frac{r_1 + r_2}{x} \times (r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{(r_1 + r_2)^2}{x} + 2x$$

It may be noted from equations 1and 2 that:
(I) The length of a crossed belt is more than that of an open belt, other conditions remaining the same.

(ii) The total length of a crossed belt is a function of $(r_1 + r_2)$. If the sum of the radii of two pulleys be constant, the length of the cross belt required will be also remain constant.

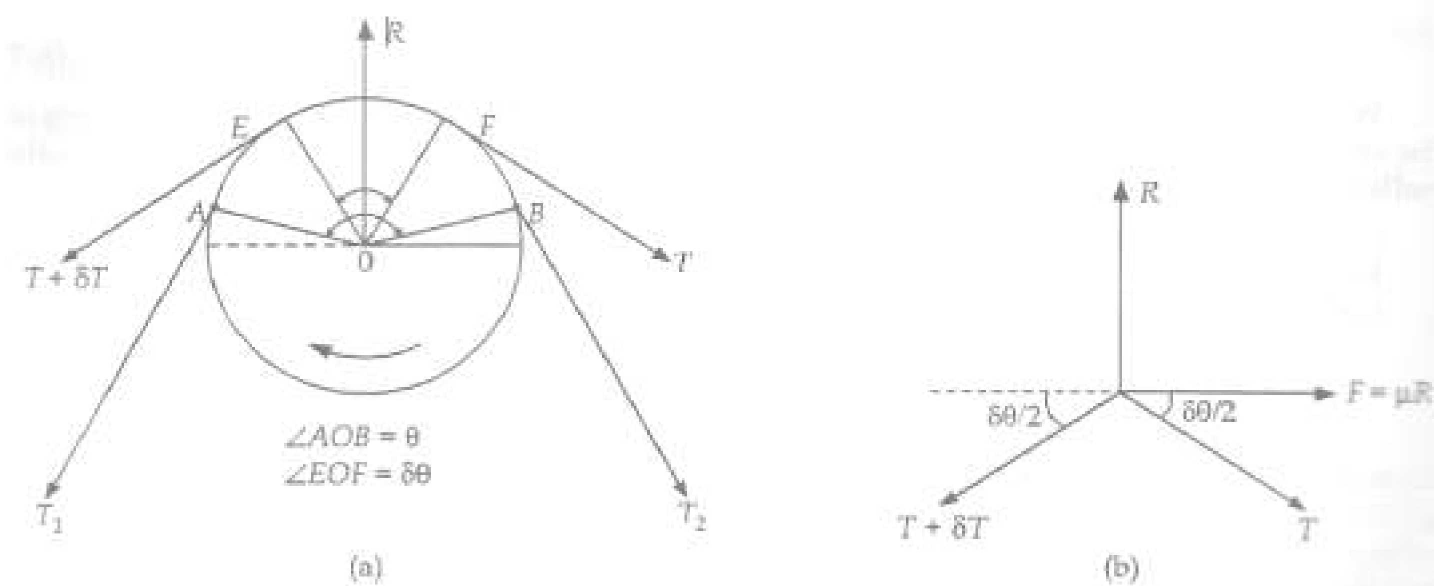
Ratio of tension

Figure shows a flexible belt resting over the flat rim of a stationary pulley. The tensions $T_1$and are such that the motion is impending Oust to take place) between the belt and the pulley. Considering the impending motion to be clockwise relative to the drum, the tension T_1 is more than T_2 is to be noted that only a part of the belt is in contact with the pulley. The angle subtended at the centre of the pulley by the position of belt in contact with it is called the *angle of contact* or the *angle of lap*

Angle of contact θ = *angle .AGB*

Let attention be focused on small element EF of the belt which subtends an angle δθ at the centre. The segment EF is acted upon by the following set of forces:

- Tension *T* in the belt acting tangentially at S,
- Tension (*T* + 8T) in the belt acting tangentially at R
- Normal reaction R exerted by the pulley rim, and
- Friction force F = μR which acts against the tendency to slip and is perpendicular to normal reaction R.



Considering equilibrium of forces in the radial (vertical) direction,

$$R = (T + \delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2}$$

For small values of $\delta\theta$; $\sin\frac{\delta\theta}{2} \rightarrow \frac{\delta\theta}{2}$

$$R = (T + \delta T)\frac{\delta\theta}{2} + T\frac{\delta\theta}{2} = T\frac{\delta\theta}{2} + \delta T\frac{\delta\theta}{2} + T\frac{\delta\theta}{2}$$

The term $\delta T\frac{\delta\theta}{2}$ is small in magnitude and can be neglected

$$R = T\frac{\delta\theta}{2} + T\frac{\delta\theta}{2} = T\,\delta\theta$$

Considering equilibrium of forces in tangential (horizontal) direction,

$$\mu R = (T + \delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2}$$

For small values of $\delta\theta$; $\cos\frac{\delta\theta}{2} \rightarrow 1$

$$\mu R = (T + \delta T) - T = \delta T$$

$$R = \frac{\delta T}{\mu}$$

From expressions (*i*) and (*ii*)

$$T\,\delta\theta = \frac{\delta T}{\mu}$$

Separating the variables and integrating between the limit $T = T_2$ at $\theta = 0$ and $T = T_1$ at $\theta = \theta$, we get:

$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_{0}^{\theta} \delta\theta$$

$$\log_e \frac{T_1}{T_2} = \mu\theta \quad \therefore \quad \frac{T_1}{T_2} = e^{\mu\theta}$$

When two pulley of unequal diameters are connected by open belt drive, the slip occurs first on

the smaller pulley where the force of friction is less. Accordingly, the angle of contact on smaller pulley is taken into account while using the above equation.

Power Transmitted by Belt Drive

The power transmitted by the belt depends on the tension on the two sides and the belt speed.
Let T_1 be the tension on the tight side in '*N*'
T_2 be the tension on the slack side in '*N*', and
V be the speed of the belt in m/sec.

Then power transmitted by the belt is given by

$$\text{Power } P = (T_1 - T_2)\,V \text{ Watt}$$

$$= \frac{(T_1 - T_2)\,V}{1000}\ \text{kW} \qquad \ldots (3.8)$$

or,

$$P = \frac{T_1\left(1 - \frac{T_2}{T_1}\right)V}{1000}\ \text{kW}$$

If belt is on the point of slipping.

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\therefore \qquad P = \frac{T_1\,(1 - e^{-\mu\theta})\,V}{1000}\ \text{kW} \qquad \ldots (3.9)$$

The maximum tension T_1 depends on the capacity of the belt to withstand force. If allowable stress in the belt is 'σ_t' in 'Pa', i.e. N/m^2, then

$$T_1 = (\sigma_t \times t \times b)\ \text{N} \qquad \ldots (3.10)$$

where t is thickness of the belt in 'm' and b is width of the belt also in m.

The above equations can also be used to determine 'b' for given power and speed.

Tension due to Centrifugal Forces

The belt has mass and as it rotates along with the pulley it is subjected to centrifugal forces. If we assume that no power is being transmitted and pulleys are rotating, the centrifugal force will tend to pull the belt as shown in Figure 3.14(b) and, thereby, a tension in the belt called centrifugal tension will be introduced.

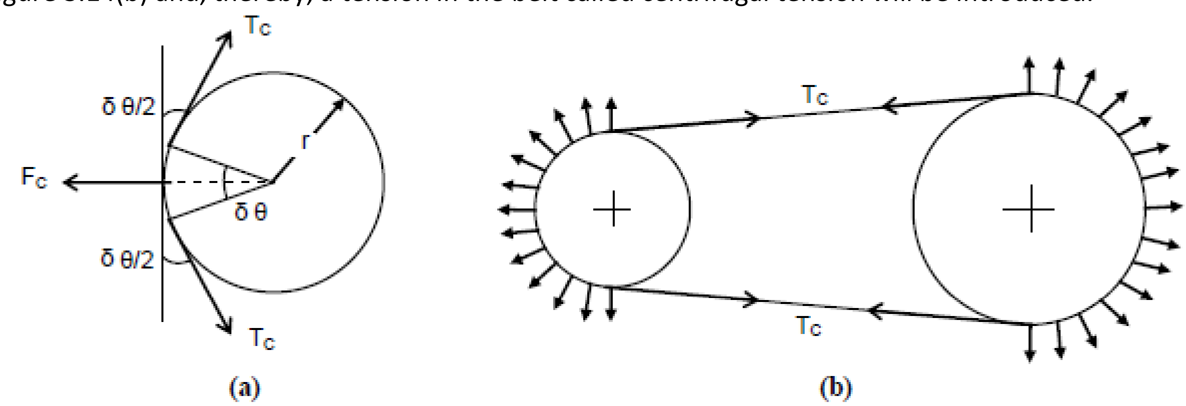


Figure 3.14 : Tension due to Centrifugal Foces

Let 'T_C' be the centrifugal tension due to centrifugal force.
Let us consider a small element which subtends an angle $\delta\theta$ at the centre of the pulley.

Let '*m*' be the mass of the belt per unit length of the belt in 'kg/m'.

The centrifugal force 'F_c' on the element will be given by

$$F_C = (r\ \delta\theta\ m) \times \frac{V^2}{r}$$

where V is speed of the belt in m/sec. and r is the radius of pulley in 'm'.

Resolving the forces on the element normal to the tangent

$$F_C - 2T_C \sin\frac{\delta\theta}{2} = 0$$

Since $\delta\theta$ is very small.

$\therefore$ $$\sin\frac{\delta\theta}{2} \approx \frac{\delta\theta}{2}$$

or, $$F_C - 2T_C\frac{\delta\theta}{2} = 0$$

or, $$F_C = T_C\ \delta\theta$$

Substituting for F_C

$$\frac{m\,V^2}{r}\,r\ \delta\theta = T_C\ \delta\theta$$

or, $$T_C = m\,V^2 \qquad \text{. . . (3.11)}$$

Therefore, considering the effect of the centrifugal tension, the belt tension on the tight side when power is transmitted is given by

Tension of tight side $T_t = T_1 + T_C$ and tension on the slack side $T_s = T_2 + T_C$.

The centrifugal tension has an effect on the power transmitted because maximum tension can be only T_t which is

$$T_t = \sigma_t \times t \times b$$

$\therefore$ $$T_1 = \sigma_t \times t \times b - m\,V^2$$

Initial Tension

When a belt is mounted on the pulley some amount of initial tension say 'T_0' is provided in the belt, otherwise power transmission is not possible because a loose belt cannot sustain difference in the tension and no power can be transmitted. When the drive is stationary the total tension on both sides will be '2 T_0'.
When belt drive is transmitting power the total tension on both sides will be ($T_1 + T_2$), where T_1 is tension on tight side, and T_2 is tension on the slack side.

If effect of centrifugal tension is neglected.

$$2T_0 = T_1 + T_2$$

or, $$T_0 = \frac{T_1 + T_2}{2}$$

If effect of centrifugal tension is considered, then

$$T_0 = T_t + T_s = T_1 + T_2 + 2T_C$$

or, $$T_0 = \frac{T_1 + T_2}{2} + T_C$$

Maximum Power Transmitted

The power transmitted depends on the tension 'T_1', angle of lap □, coefficient of friction '□' and belt speed 'V'. For a given belt drive, the maximum tension (T_t), angle of lap and coefficient of friction shall remain constant provided that (a) the tension on tight side, i.e. maximum tension should be equal to the maximum permissible value for the belt, and

(b) the belt should be on the point of slipping.

Therefore, Power $P = T_1 \, (1 - e^{-\mu\theta}) \, V$

Since, $T_1 = T_t + T_c$

or, $P = (T_t - T_c)\,(1 - e^{-\mu\theta})\,V$

or, $P = (T_t - m\,V^2)\,(1 - e^{-\mu\theta})\,V$

For maximum power transmitted

$\therefore$ $$\frac{dP}{dV} = (T_t - 3m\,V^2)\,(1 - e^{-\mu\theta})$$

or, $T_t - 3m\,V^2 = 0$

or, $T_t - 3T_c = 0$

or, $$T_c = \frac{T_t}{3}$$

or, $$m\,V^2 = \frac{T_t}{3}$$

Also, $$V = \sqrt{\frac{T_t}{3m}} \qquad \text{. . . (3.13)}$$

At the belt speed given by the Eq. (3.13) the power transmitted by the belt drive shall be maximum.

Example 3.2

An open flat belt drive is required to transmit 20 kW. The diameter of one of the pulleys is 150 cm having speed equal to 300 rpm. The minimum angle of contact may be taken as 170°. The permissible stress in the belt may be taken as 300 N/cm^2. The coefficient of friction between belt and pulley surface is 0.3. Determine

(a) width of the belt neglecting effect of centrifugal tension for belt thickness equal to 8 mm.

(b) width of belt considering the effect of centrifugal tension for the thickness equal to that in (a). The density of the belt material is 1.0 gm/cm^3.

Solution

Given that Power transmitted (p) = 20 kW

Diameter of pulley (d) = 150 cm = 1.5 m

Speed of the belt (N) = 300 rpm

Angle of lap (θ) $= 170^o = \frac{170}{180}\pi = 2.387$ radian

Coefficient of friction (μ) = 0.3

Permissible stress (σ) = 300 N/cm^2

(a) Thickness of the belt (t) = 8 mm = 0.8 cm

Let higher tension be 'T_1' and lower tension be 'T_2'.

$$\therefore \quad \frac{T_1}{T_2} = e^{\mu\theta} = e^{0.3 \times 2.387} = 2.53$$

The maximum tension 'T_1' is controlled by the permissible stress.

$$T_1 = \sigma\, b\, t = 300 \times \frac{b}{10} \times 0.8 = 24b \text{ N}$$

Here b is in mm

Therefore, $T_2 = \frac{T_1}{2.53} = \frac{24b}{2.53}$ N

Velocity of belt $V = \frac{2\pi N}{60} \times \frac{d}{2} = \frac{2\pi \times 300}{60} \times \frac{1.5}{2} = 23.5$ m/s

$$\therefore \quad \text{Power transmitted } p = (T_1 - T_2)\, V = \left(24b - \frac{24b}{2.53}\right) \times \frac{23.5}{1000} \text{ kW}$$

$$= 24b\left(1 - \frac{1}{2.53}\right) \times \frac{23.5}{1000} = \frac{347.3b}{1000}$$

Since $P = 20$ kW

$$\therefore \quad \frac{347.3b}{1000} = 20$$

or, $$b = \frac{20 \times 1000}{347.3} = 36.4 \text{ mm}$$

(b) The density of the belt material $\rho = 1$ gm/cm^3

Mass of the belt material/length, $m = \rho\, b\, t \times 1$ metre

$$= \frac{1}{1000} \times \frac{b}{10} \times 0.8 \times 100 = 0.8 \times 10^{-2}\, b \text{ kg/m}$$

$$= 8b \times 10^{-3} \text{ kg/m}$$

$\therefore$ Centrifugal tension 'T_C' $= m\,V^2$

or, $$T_C = 8b \times 10^{-3} \times (23.5)^2 = 4.418b \text{ N}$$

Maximum tension (T_{max}) = $24b$ N

$\therefore$ $$T_1 = T_{max} - T_C = 24b - 4.418b = 19.58b$$

Power transmitted $P = T_1 \left(1 - \frac{1}{e^{\mu\theta}}\right) V$

$$= 19.58b \left(1 - \frac{1}{2.53}\right) \times \frac{23.5}{1000} = \frac{460.177b}{1000}$$

Also $P = 20$ kW

$\therefore$ $$\frac{460.177b}{1000} = 20$$

or, $b = 45.4$ mm

The effect of the centrifugal tension increases the width of the belt required.

Example 3.3

An open belt drive is required to transmit 15 kW from a motor running at 740 rpm. The diameter of the motor pulley is 30 cm. The driven pulley runs at 300 rpm and is mounted on a shaft which is 3 metres away from the driving shaft. Density of the leather belt is 0.1 gm/cm^3. Allowable stress for the belt material is 250 N/cm^2. If coefficient of friction between the belt and pulley is 0.3, determine width of the belt required. The thickness of the belt is 9.75 mm.

Solution

Given data :

Power transmitted (P) = 15 kW

Speed of motor pulley (N_1) = 740 rpm

Diameter of motor pulley (d_1) = 30 cm

Speed of driven pulley (N_2) = 300 rpm

Distance between shaft axes (C) = 3 m

Density of the belt material (ρ) = 0.1 gm/cm^3

Allowable stress (σ) = 250 N/cm^2

Coefficient of friction (μ) = 0.3

Let the diameter of the driven pulley be 'd_2'

$\therefore \quad N_1 d_1 = N_2 d_2$

$\therefore \quad d_2 = \frac{N_1 d_1}{N_2} = \frac{740 \times 30}{300} = 74 \text{ cm}$

$\sin\beta = \frac{d_2 - d_1}{2C} \quad \therefore \quad \beta = \sin^{-1}\frac{74-30}{2 \times 300}$

or, $\quad \beta = 0.0734$ radian

$\theta = \pi - 2\beta = 2.94$ rad

Mass of belt 'm' = $\rho\, b\, t$ × one metre length

$$= \frac{0.1}{1000} \times \frac{b}{10} \times \frac{9.75}{10} \times 100$$

where 'b' is width of the belt in 'mm'

or, $\quad m = 0.975 \times 10^{-3}\, b$ kg/m

$$T_{max} = 250 \times \frac{b}{10} \times \frac{9.75}{10} = 24.375\, b \text{ N}$$

Active tension 'T' = $T_{max} - T_C$

Velocity of belt $V = \frac{2\pi N_1}{60}\frac{d_1}{2}$

$$= \frac{\pi \times 740}{60} \times \frac{30}{100}$$

or, $\quad V = 11.62$ m/s

$$T_C = m V^2 = 0.975 \times 10^{-3}\, b \times (11.62)^2$$

$$= 0.132\, b \text{ N}$$

$\therefore \quad T_1 = 24.375\, b - 0.132\, b = 24.243\, b$

Power transmitted $P = T_1\left(1 - \frac{1}{e^{\mu\theta}}\right)V$

$$e^{\mu\theta} = e^{0.3 \times 2.94} = 2.47$$

$$P = 24.243\left(1 - \frac{1}{2.47}\right) \times \frac{11.62}{1000} = \frac{165\,b}{1000}$$

$$\frac{165\,b}{100} = 15 \quad \text{or} \quad b = 91 \text{ mm}$$

Example 3.4

An open belt drive has two pulleys having diameters 1.2 m and 0.5 m. The pulley shafts are parallel to each other with axes 4 m apart. The mass of the belt is 1 kg per metre length. The tension is not allowed to exceed 2000 N. The larger pulley is driving pulley and it rotates at 200 rpm. Speed of the driven pulley is 450 rpm due to the belt slip. The coefficient of the friction is 0.3. Determine

(a) power transmitted,

(b) power lost in friction, and

(c) efficiency of the drive.

Solution

Data given :

Diameter of driver pulley (d_1) = 1.2 m

Diameter of driven pulley (d_2) = 0.5 m

Centre distance (C) = 4 m

Mass of belt (m) = 1 kg/m

Maximum tension (T_{max}) = 2000 N

Speed of driver pulley (N_1) = 200 rpm

Speed of driven pulley (N_2) = 450 rpm

Coefficient of friction (μ) = 0.3

(a) $$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 200}{60} = 20.93 \text{ r/s}$$

$$\omega_2 = \frac{2\pi N_2}{60} = \frac{2\pi\, 450}{60} = 47.1 \text{ r/s}$$

Velocity of the belt (V) $= 20.93 \times \frac{1.2}{2} = 12.56$ m/s

Centrifugal tension (T_C) $= m\,V^2 = 1 \times (12.56)^2 = 157.75$ N

Active tension on tight side (T_1) $= T_{max} - T_C$

or, $T_1 = 2000 - 157.75 = 1842.25$ N

$$\sin\beta = \frac{d_1 - d_2}{2C} = \frac{1.2 - 0.5}{2 \times 4} = 0.0875$$

or, $\beta = 5.015°$

$$\theta = 180 - 2\beta = 180 - 2 \times 5.015 = 169.985°$$

or, $$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.3 \times \frac{169.985}{180}} = 2.43$$

$$\text{Power transmitted } (P) = T_1\left(1-\frac{1}{2.43}\right)\times 12.56$$

$$= 1842.25\left(1-\frac{1}{2.43}\right)\frac{12.56}{1000}\ \text{kW}$$

$$= 13.67\ \text{kW}$$

(b) $$\text{Power output} = T_1\left(1-\frac{1}{2.43}\right)\times\frac{\omega_2 d_2}{2}\ \text{W}$$

$$= 1842.25\left(1-\frac{1}{2.43}\right)\times\frac{47.1\times 0.5}{2\times 1000} = 12.2\ \text{kW}$$

$\therefore$ Power lost in friction = 13.67 − 12.2 = 1.47 kW

(c) $$\text{Efficiency of the drive} = \frac{\text{Power transmitted}}{\text{Power input}} = \frac{12.2}{13.67} = 0.89 \text{ or } 89\%.$$

Example 3.5

A leather belt is mounted on two pulleys. The larger pulley has diameter equal to 1.2 m and rotates at speed equal to 25 rad/s. The angle of lap is 150°. The maximum permissible tension in the belt is 1200 N. The coefficient of friction between the belt and pulley is 0.25. Determine the maximum power which can be transmitted by the belt if initial tension in the belt lies between 800 N and 960 N.

Solution

Given data :

Diameter of larger pulley (d_1) = 1.2 m

Speed of larger pulley ω_1 = 25 rad/s

Speed of smaller pulley ω_2 = 50 rad/s

Angle of lap (θ) = 150°

Initial tension (T_0) = 800 to 960 N

Let the effect of centrifugal tension be negligible.

The maximum tension (T_1) = 1200 N

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.25\times\frac{150}{180}\times\pi} = 1.924$$

$$T_2 = \frac{T_1}{1.924} = \frac{1200}{1.924} = 623.6\ \text{N}$$

$$T_0 = \frac{T_1+T_2}{2} = \frac{1200+623.6}{2} = 911.8\ \text{N}$$

Maximum power transmitted (P_{max}) = ($T_1 - T_2$) V

$$\text{Velocity of belt } (V) = \frac{d_1}{2}\,\omega_1 = \frac{1.2}{2}\times 25$$

$$V = 15\ \text{m/s}$$

$$\therefore \quad P_{max} = (1200-623.6)\,V = (1200-623.6)\,15$$

$$= 8646\ \text{W} \quad \text{or} \quad 8.646\ \text{kW}$$

Gear Drive

A gear is a wheel provided with teeth which mesh with the teeth on another wheel, or on to a rack, so as to give a positive transmission of motion from one component to another.

They are commonly used for power transmission or for changing power speed ratio in a power system but when they are not too far apart and when a constant velocity ratio is desired.

Advantages and Disadvantages of Gear Drive

The following are the advantages and disadvantages of the gear drive as compared to belt, rope and chain drives:

Advantages

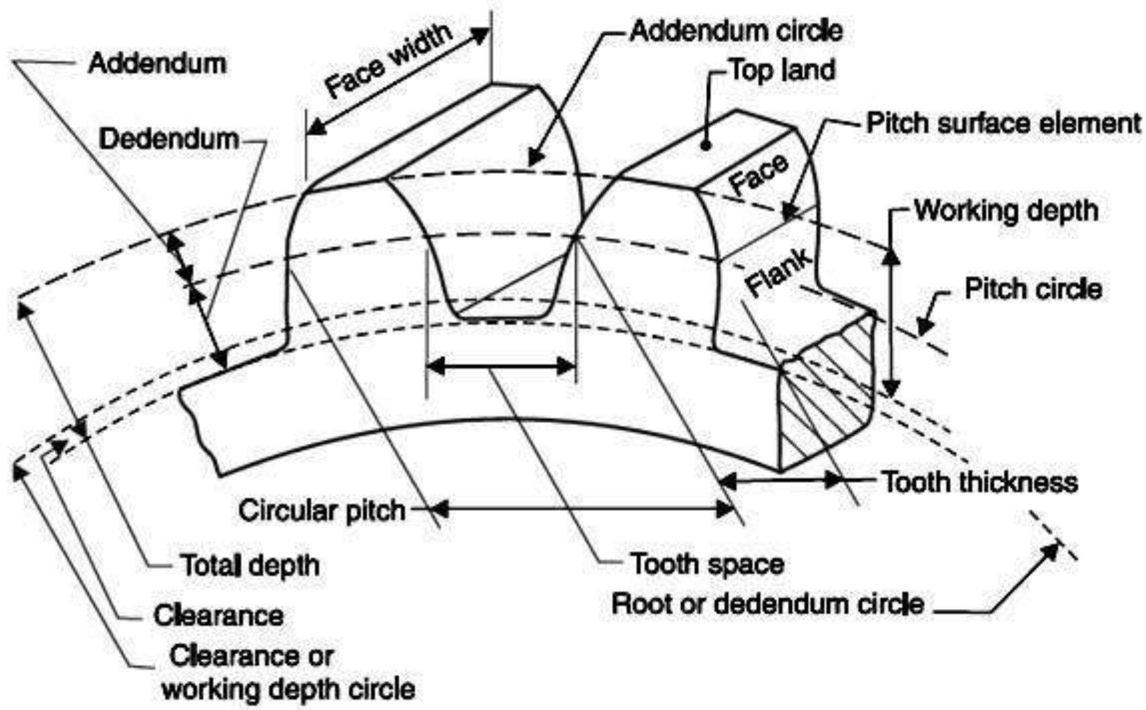
1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It has high efficiency.
4. It has reliable service.
5. It has compact layout.

Disadvantages

1. The manufacture of gears require special tools and equipment.
2. The error in cutting teeth may cause vibrations and noise during operation

Definitions

There are several notation which are given below:



1. Pitch circle. It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

2. Pitch circle diameter. It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as **pitch diameter**.

3. Pitch point. It is a common point of contact between two pitch circles.

4. Pitch surface. It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

5. Pressure angle or angle of obliquity. It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by φ. The standard pressure angles are 14½ ° and 20°.

6. Addendum. It is the radial distance of a tooth from the pitch circle to the top of the tooth.

7. Dedendum. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

8. Addendum circle. It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

9. Dedendum circle. It is the circle drawn through the bottom of the teeth. It is also called root circle.

Note: Root circle diameter = Pitch circle diameter × cos φ, where φ is the pressure angle.

10. Circular pitch. It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by pc.

Mathematically,

Circular pitch, $P_c = \Pi D/T$

where D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

A little consideration will show that the two gears will mesh together correctly, if the two wheels have the same circular pitch.

Note: If D1 and D2 are the diameters of the two meshing gears having the teeth T1 and T2 respectively, then for them to mesh correctly,

$\mathbf{P_C} = \Pi D_1/T = \Pi D_2/T$

11. Diametral pitch. It is the ratio of number of teeth to the pitch circle diameter in millimetres. It is denoted by pd . Mathematically,

Diametral pitch, $P_d = T/D = \Pi/\mathbf{P_C}$

where T = Number of teeth, and

D = Pitch circle diameter.

12. Module. It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m. mathematically,

Module, m = D /T

Note : The recommended series of modules in Indian Standard are 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, and 20. The modules 1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14 and 18 are of second choice.

13. Clearance. It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as **clearance circle**.

14. Total depth. It is the radial distance between the addendum and the dedendum circles of a gear. It is equal to the sum of the addendum and dedendum.

15. Working depth. It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.
16. Tooth thickness. It is the width of the tooth measured along the pitch circle.
17. Tooth space . It is the width of space between the two adjacent teeth measured along the pitch circle.

18. Backlash. It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.

19. Face of tooth. It is the surface of the gear tooth above the pitch surface.
20. Flank of tooth. It is the surface of the gear tooth below the pitch surface.
21. Top land. It is the surface of the top of the tooth.
22. Face width. It is the width of the gear tooth measured parallel to its axis.
23. Profile. It is the curve formed by the face and flank of the tooth.
24. Fillet radius. It is the radius that connects the root circle to the profile of the tooth.
25. Path of contact. It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.
26. ***Length of the path of contact**. It is the length of the common normal cut-off by the addendum circles of the wheel and pinion.
27. ** **Arc of contact**. It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, i.e.
(a) Arc of approach. It is the portion of the path of contact from the beginning of the engagement to the pitch point.
(b) Arc of recess. It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.
Note : The ratio of the length of arc of contact to the circular pitch is known as **contact ratio** i.e. number of pairs of teeth in contact.

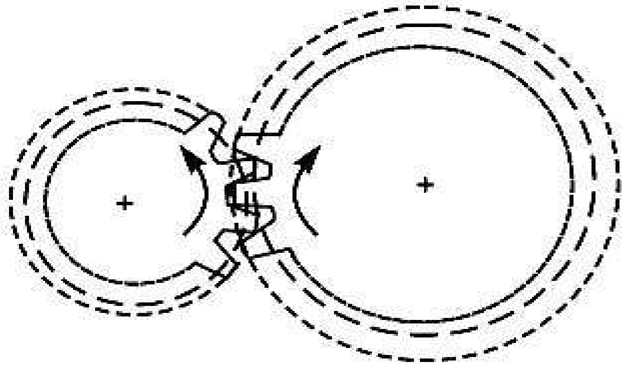
Types of gear

There are mainly five types of gear

1. Spur gear
2. Helical gear
3. Bevel gear
4. Worm gear
5. Rack & pinion gear

Spur gear

Spur gears or straight-cut gears are the simplest type of gear. They consist of a cylinder or disk, and with the teeth projecting radially, and although they are not straight-sided in form, the edge of each tooth thus is straight and aligned parallel to the axis of rotation. These gears can be meshed together correctly only if they are fitted to parallel axles. The arrangement is known as **spur gearing**.

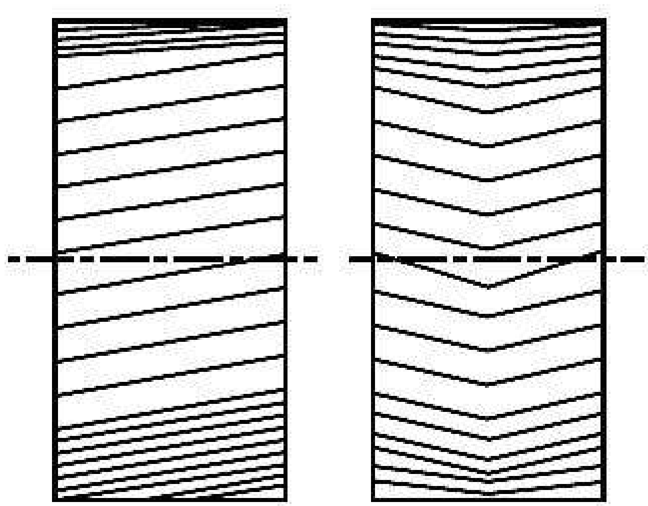


Helical gear

The teeth of helical gear are inclined to the axis In this type of gear. This ensures smooth action & more accurate maintenance of velocity ratio.

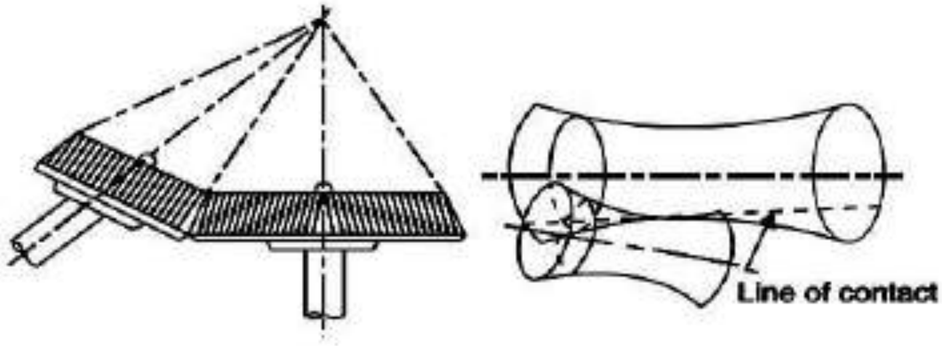
The disadvantage is that the inclination of the teeth sets up a lateral thrust. A method of neutralizing this lateral or axial thrust is to use double helical gears (also known as herring bone gear)

(a)Single helical gear (b) double helical gear



Bevel gear

The two non-parallel or intersecting, but coplanar shafts connected by gears are called **bevel gears** and the arrangement is known as **bevel gearing**. The bevel gears, like spur gears, may also have their teeth inclined to the face of the bevel, in which case they are known as **helical bevel gears**. The two non-intersecting and non-parallel i.e. non-coplanar shaft connected by gears are called **skew bevel gears** or **spiral gears** and the arrangement is known as **skew bevel gearing** or **spiral gearing**. This type of gearing also has a line contact, the rotation of which about the axes generates the two pitch surfaces known as **hyperboloids**.



(c) Bevel gear. (d) Spiral gear.

Worm gear

They connect non-parallel, non-intersecting shafts which are usually at right angles. One of the gear is called „worm‟. It is essential part of a screw, meshing with the teeth on a gear wheel, called the "worm wheel". The gear ratio is the ratio of number of teeth on the wheel to the number of thread on the worm. Its advantage is that it gives high gear ratio which are easily obtained & also smooth & quiet.

Rack & pinion gear

A rack is a spur gear of infinite diameter, thus it assumes the shape of a straight gear. The rack is generally used with a pinion to convert rotary motion into rectilinear motion

Types of Gear Trains

Two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called **gear train** or **train of toothed wheels.** The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts. A gear train may consist of spur, bevel or spiral gears.

The different types of gear trains, depending upon the arrangement of wheels:

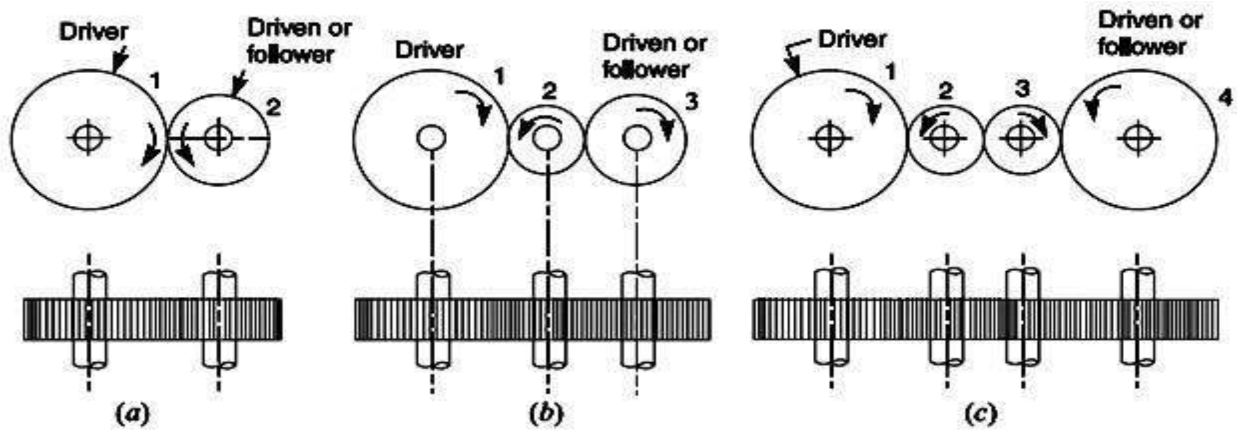
1. Simple gear train,
2. Compound gear train,
3. Reverted gear train, and
4. Epicyclic gear train.

In the first three types of gear trains, the axes of the shafts over which the gears are mounted are **fixed** relative to each other. But in case of epicyclic gear trains, the axes of the shafts on which the gears are mounted may **move** relative to a fixed axis.

Simple gear train

When there is only one gear on each shaft, as shown in Figure, it is known as **simple gear train.** The gears are represented by their pitch circles. When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in Fig.(a).Since the gear 1 drives the gear 2, therefore gear 1 is called the **driver** and the gear 2 is called the **driven** or **follower.** It may be noted

that the motion of the driven gear is opposite to the motion of driving gear.



Let
N1 = Speed of gear 1(or driver) in r.p.m.,
N2 = Speed of gear 2 (or driven or follower) in r.p.m.,
T1 = Number of teeth on gear 1, and

T2 = Number of teeth on gear 2.

Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore
Speed ratio = $N_1/N_2=T_2/T_1$
It may be noted that ratio of the speed of the driven or follower to the speed of the driver is known as **train value** of the gear train. Mathematically,
Train value = $N_2/N_1=T_1/T_2$
From above, we see that the train value is the **reciprocal** of speed ratio.
Sometimes, the distance between the two gears is **large**. The motion from one gear to another, in such a case, may be transmitted by either of the following two methods:
1. By providing the large sized gear, or
2. By providing one or more intermediate gears.
A little consideration will show that the former method (i.e. providing large sized gears) is veryinconvenient and uneconomical method ; whereas the latter method (i.e. providing one or more intermediate gear) is very convenient and economical. It may be noted that when the number of intermediate gears are **odd,** the motion of both the gears (i.e. driver and driven or follower) is **like** as shown in Fig. (b). But if the number of intermediate gears are **even,** the motion of the driven or follower will be in the opposite direction of the driver as shown in Fig. (c).
Now consider a simple train of gears with one intermediate gear as shown in Fig. (b).
Let
N1 = Speed of driver in r.p.m.,

N2 = Speed of intermediate gear in r.p.m.,
N3 = Speed of driven or follower in r.p.m.,
T1 = Number of teeth on driver,
T2 = Number of teeth on intermediate gear, and
T3 = Number of teeth on driven or follower.

Since the driving gear 1 is in mesh with the intermediate gear 2, therefore speed ratio for these two gears is

$N_1/N_2=T_2/T_1$ ...**(i)**

Similarly, as the intermediate gear 2 is in mesh with the driven gear 3, therefore speed ratio for these two gears is

$$N_2/N_3=T_3/T_2 \quad \textbf{...(ii)}$$

The speed ratio of the gear train as shown in Fig. 13.1 (b) is obtained by multiplying the equations **(i)** and **(ii).**

$$N_1/N_2 \times N_2/N_3=T_2/T_1 \times T_3/T_2$$

Or

$$N_1/N_3 = T_1/T_3$$

i.e.

$$\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$

and

$$\text{Train value} = \frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$

Similarly, it can be proved that the above equation holds good even if there are any number of intermediate gears. From above, we see that the speed ratio and the train value, in a simple train of gears, is independent of the size and number of intermediate gears. These intermediate gears are called **idle gears,** as they do not affect the speed ratio or train value of the system. The idle gears are used for the following two purposes

1. To connect gears where a large centre distance is required, and
2. To obtain the desired direction of motion of the driven gear (i.e. clockwise or anticlockwise).

Compound Gear Train

When there are more than one gear on a shaft, as shown in Fig. 2. it is called a **compound train of gear** . The idle gears, in a simple train of gears do not effect the speed ratio of the system. But these gears are useful in bridging over the space between the driver and the driven.

But whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great (or much less) speed ratio is required, then the advantage of intermediate gears is intensified by providing compound gears on intermediate shafts. In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed. One of these two gears meshes with the driver and the other with the driven or follower attached to the next shaft as shown in Fig. 2.

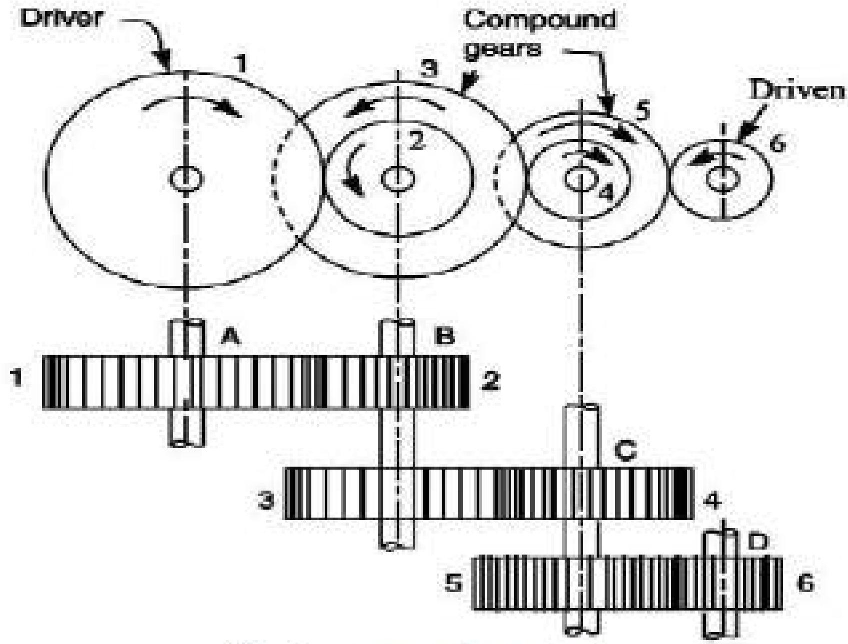


Fig 2 compound gear train

In a compound train of gears, as shown in Fig. 2, the gear 1 is the driving gear mounted on shaft A, gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.

Let

N1 = Speed of driving gear 1,
T1 = Number of teeth on driving gear 1,
N2 ,N3 ..., N6 = Speed of respective gears in r.p.m., and
T2 ,T3..., T6 = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$N_1/N_2=T_2/T_1 \qquad \text{...(i)}$$

Similarly, for gears 3 and 4, speed ratio is

$$N_3/N_4=T_4/T_3 \qquad \text{...(ii)}$$

and for gears 5 and 6, speed ratio is

$$N_5/N_6=T_6/T_5 \qquad \text{...(iii)}$$

The speed ratio of compound gear train is obtained by multiplying the equations **(i)**, **(ii)** and **(iii)**,

$$\therefore \quad \frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5} \quad \text{or} \quad {}^*\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

i.e.

$$\text{Speed ratio} = \frac{\text{Speed of the first driver}}{\text{Speed of the last driven or follower}} = \frac{\text{Product of the number of teeth on the drivens}}{\text{Product of the number of teeth on the drivers}}$$

and

$$\text{Train value} = \frac{\text{Speed of the last driven or follower}}{\text{Speed of the first driver}} = \frac{\text{Product of the number of teeth on the drivers}}{\text{Product of the number of teeth on the drivens}}$$

The advantage of a compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears. If a simple gear train is used to give a large speed reduction, the last gear has to be very large. Usually for a speed reduction in excess of 7 to 1, a simple train is not used and a compound train or worm gearing is employed.

Note: The gears which mesh must have the same circular pitch or module. Thus gears 1 and 2 must have the same module as they mesh together. Similarly gears 3 and 4, and gears 5 and 6 must have the same module.

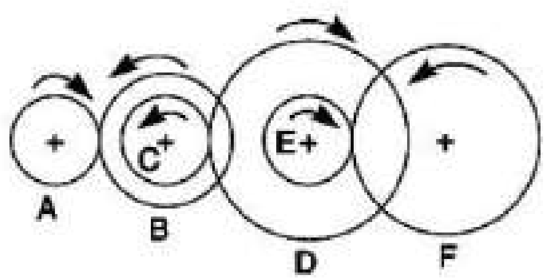


Fig 3

Reverted Gear Train

When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial, then the gear train is known as **reverted gear train** as shown in Fig. a. We see that gear 1 (i.e. first driver) drives the gear 2 (i.e. first driven or follower) in the opposite direction.

Since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound gear and the gear 3 will rotate in the same direction as that of gear 2. The gear 3 (which is now the second driver) drives the gear 4 (i.e. the last driven or follower) in the same direction as that of gear 1. Thus we see that in a reverted gear train, the motion of the first gear and the last gear is **like.**

Let T_1 = Number of teeth on gear 1,
r_1 = Pitch circle radius of gear 1, and
N_1 = Speed of gear 1 in r.p.m.

Similarly,

T_2, T_3, T_4 = Number of teeth on respective gears,
r_2, r_3, r_4 = Pitch circle radii of respective gears, and
N_2, N_3, N_4 = Speed of respective gears in r.p.m.

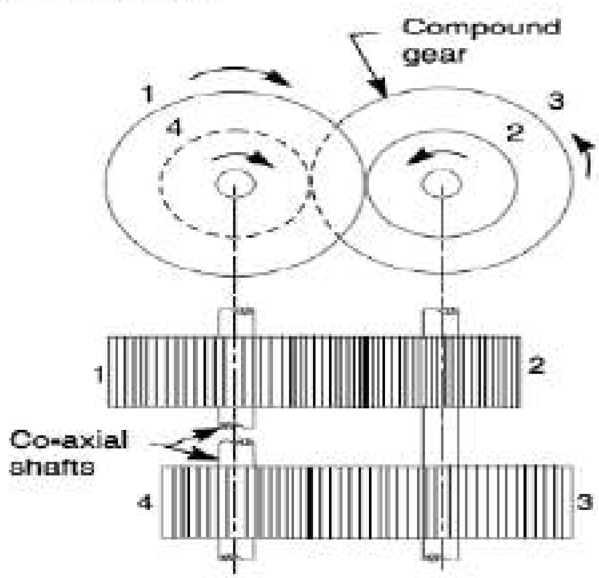
Since the distance between the centres of the shafts of gears 1 and 2 as well as gears 3 and 4 is same, therefore

$$r_1 + r_2 = r_3 + r_4 \quad \text{...(i)}$$

Also, the circular pitch or module of all the gears is assumed to be same, therefore number of teeth on each gear is directly proportional to its circumference or radius.

$$T_1 + T_2 = T_3 + T_4 \quad \text{...(ii)}$$

and Product of number of teeth on drivens



Reverted gear train.

and $$\text{Speed ratio} = \frac{\text{Product of number of teeth on drivens}}{\text{Product of number of teeth on drivers}}$$

or $$\frac{N_1}{N_A} = \frac{T_2 \times T_4}{T_1 \times T_3} \quad \text{... } (iii)$$

From equations **(i), (ii)** and **(iii),** we can determine the number of teeth on each gear for the given centre distance, speed ratio and module only when the number of teeth on one gear is chosen arbitrarily.

The reverted gear trains are used in automotive transmissions, lathe back gears, industrial speed reducers, and in clocks (where the minute and hour hand shafts are co-axial).

Epicyclic Gear Train

We have already discussed that in an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis. A simple epicyclic gear train is shown in Fig. b, where a gear A and the arm C have a common axis at O1 about which they can rotate. The gear B meshes with gear A and has its axis on the arm at O2, about which the gear B can rotate. If the arm is fixed, the gear train is simple and gear A can drive gear B or **vice- versa**, but if gear A is fixed and the arm is rotated about the axis of gear A (i.e. O1), then the gear B is forced to rotate **upon** and **around** gear A. Such a motion is called **epicyclic** and the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as **epicyclic gear trains (epi.** means upon and **cyclic** means around). The epicyclic gear trains may be **simple** or **compound.**

The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space. The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.

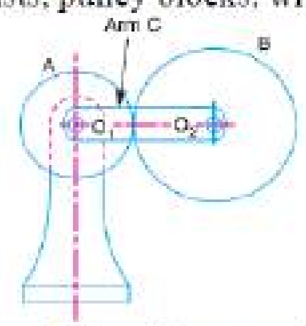


Fig b Epicyclic gear train.