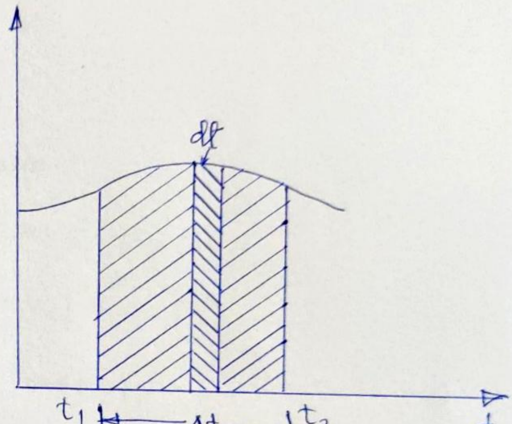
Impulse and Momentum

Impulse of a force:

When a large force acts over a short period of time that force is called an impulsive force.

The impulse of force F acting over a time interval for t_1 to t_2 is defined by the integral,

$$I = \int_{t_1}^{t_2} F\,dt$$

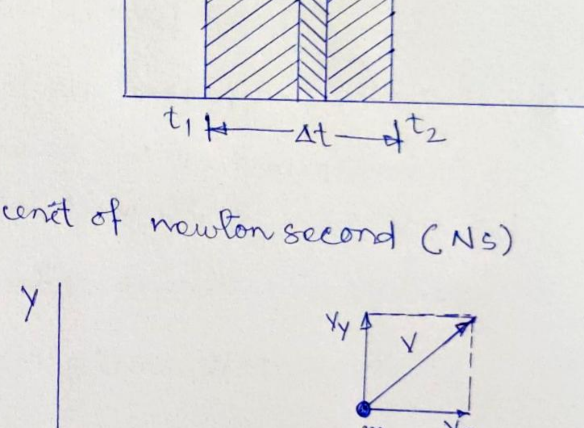


The impulse of a force, therefore, can be visualised as the area under the force vs. time diagram as shown in fig.

$$I = F_{avg.} \times \Delta t$$

Impulse of a force is a vector quanity and has the unit of newton second (NS)

Momentum:



Consider the motion of a particle of mass m acted upon by a force F.

The equation of motion of the particle in the x and y direction are.

$$F_x = ma_x \quad \text{and} \quad F_y = ma_y$$

or $$F_x = \frac{m\,dV_x}{dt} \quad \text{and} \quad F_y = \frac{m\,dV_y}{dt}$$

or $$F_x = \frac{d}{dt}(mV_x) \quad \text{and} \quad F_y = \frac{d}{dt}(mV_y) \quad \text{— (1)}$$

A single equation in the vector form can be written as,

$$F = \frac{d}{dt}(mv) \quad \text{—— (2)}$$

which states that the force F acting on the particle is equal to the rate of change of momentum of the particle.

The vector mv is called the momentum or the linear momentum. It has the same direction as the velocity of the particle. Unit of momentum is

$$mv = kg\left(\frac{m}{s}\right) = kg\left(\frac{m}{s^2}\right)s = Ns$$

Principle of Impulse and Momentum:

Multiplying both the sides of the equations (1) by dt,

$$F_x\,dt = d(mv_x) \quad \text{and} \quad F_y\,dt = d(mv_y) \quad \text{—— (3)}$$

where, $F_x\,dt$ is the impulse of the force F_x.

$d(mv_x)$ is the differential change in the momentum of the particle in the x-direction in time dt.

$F_y\,dt$ and $d(mv_y)$ denote similar quantities in y-direction.

Integrating equations (3) from a time t_1 to a time t_2

$$\left.\begin{aligned}\int_{t_1}^{t_2} F_x\,dt &= (mv_x)_2 - (mv_y)_1 \\ \int_{t_1}^{t_2} F_y\,dt &= (mv_y)_2 - (mv_y)_1\end{aligned}\right\} \quad \text{— (4)}$$

The above two equation can be combined into a single vector equation as

$$\int_{t_1}^{t_2} F\,dt = mv_2 - mv_1 \qquad \text{if } t_1 = 0, \text{ and } t_2 = t$$

$$\boxed{mv_2 - mv_1 = \int_0^t F\,dt} \quad \text{—— (5)}$$

final momentum − Initial momentum = Impulse of the force.

Equation (5) express that the total change in the momentum of a particle during a time interval is equal to the impulse of the force acting during the same interval of time.

Conservation of momentum :

It can be observed from equation (5) that when the external force $F = 0$; then $\boxed{mv_2 = mv_1}$ — (6)

for a system of particles, eqⁿ (6) can be written as

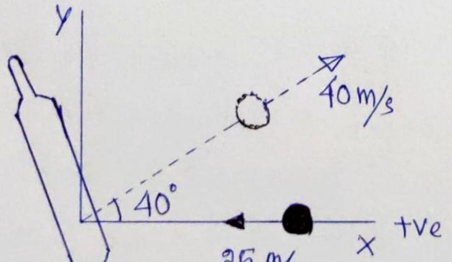
$$\boxed{\Sigma mv_2 = \Sigma mv_1} \quad — (7)$$

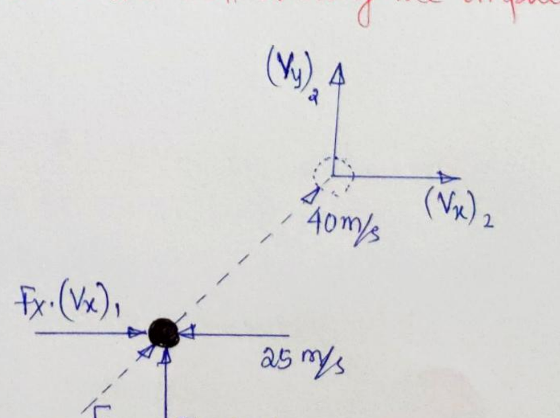
i.e. momentum of the system remains constant or is conserved.

final momentum = Initial momentum.

Q. A ball of mass 100gm is moving towards a bat with a velocity of 25 m/s as shown in fig. when hit by a bat the ball attains a velocity of 40 m/s. If the bat and the ball are in contact for a period of 0.015 s. Determine the average impulse force exerted by the bat on the ball during the impact.

Solⁿ:





Let us apply the principle of impulse and momentum to the ball in the x and y directions.

$$(mV_x)_2 - (mV_x)_1 = \int_0^t F_x\,dt$$

$m = \frac{100}{1000} = 0.1\,kg$, $\Delta t = 0.015s$, $(V_x)_1 = -25\,m/s$

$(V_x)_2 = 40\cos 40^\circ = 30.64\,m/s$

$$\int_0^t F_x\,dt = (F_x)_{avg.}(\Delta t) = 0.1(30.64) - 0.1(-25)$$

$$(F_x)_{avg.} = \frac{5.564}{0.015} = \underline{370.9\,N}$$

In Y-direction,

$$(mV_y)_2 - (mV_y)_1 = \int_0^t F_y\,dt$$

$(V_y)_1 = 0$, $(V_y)_2 = 40\sin 40 = 25.72\,m/s$

$$\int_0^t F_y\,dt = (F_y)_{avg.}(\Delta t) = 0.1(25.72) - 0.1(0)$$

$$(F_y)_{avg.} = \frac{2.572}{0.015} = \underline{171.5\,N}$$

$$F_{avg.} = \sqrt{(F_x)^2_{avg.} + (F_y)^2_{avg.}} = \sqrt{(370.9)^2 + (171.5)^2}$$

$F_{avg.} = \underline{408.6\,N}$ Ans.

Q. Recoil of Gun:

When the bullet is fired from the gun, the opposite reaction of the bullet is known as recoil of gun.
Let M = mass of the gun, V = velocity of the gun with which it recoils, m = mass of the bullet and u = velocity of the bullet.
The momentum of the bullet after explosion $= mu$
The momentum of the gun $= MV$
Then, $\boxed{MV = mu}$ conservation of momentum.

Q. A machine gun of mass 30 kg fires a bullet of mass 35 gm with a velocity of 300 m/s. find the velocity with which the machine gun will recoil.

Sol^n: Given data: mass of the gun, $M = 30$ kg

mass of the bullet, $m = 35$ gm $= 0.035$ kg and

velocity of the bullet $u = 300$ m/s

$$MV = mu \iff 30 \times V = 0.035 \times 300$$

$$V = \frac{0.035 \times 300}{30} = \underline{\underline{0.35 \text{ m/s}}} \text{ Ans.}$$