Collision of Elastic Bodies

Impact means the collision of two bodies which occure in a very short interval of time. Elasticity is the property of bodies by virtue of which they rebound after impact. An inelastic body is one which does not rebound at all. When two elastic bodies collide, they are deformed at first, and then they start spring apart because of the action of the restoring elastic forces.

Time of compression:

The time taken by two bodies in compression after the instant of collision is known as the time of compression.

Time of restitution:

Whenever two bodies (elastic) collide with each other, they tend to compress each other. Immediately after this, the two bodies attempt to regain their original shapes, due to elasticity. This process of regaining the original shape is called restitution. The time taken by two bodies to regain the original shapes after compression is known as the time of restitution.

Time of collision or period of impact:

The sum of the time of compression and the time of restitution is known as the time of collision or the period of impact.

Line of impact:

It is the line joining the centers of the two bodies with the point of contact.

Principle of conservation of Momentum

It states that "if no external forces act on a system of colliding bodies, the total momentums of the bodies before and after the collision remain the same."

Consider two bodies A and B colliding with each other. By Newton's third law, the force exerted by the body A on B must be equal and opposite to that exerted by the B on A.

Let I_1 = impulse from A to B

and I_2 = impulse from B to A

Because the total momentum remains constant, i.e.

$$I_1 = I_2$$

$$m_1 (v_1 - u_1) = -m_2 (v_2 - u_2)$$

or $$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

i.e. Sum of momentum before impact = Sum of momentum after impact

Newton's law of Collision of Elastic Bodies:

According to Newton's law of collision, "when two moving bodies collide with each other, their velocity of separation bears a constant ratio to their velocity of approach."

Mathematically, $(v_2 - v_1) = e(u_1 - u_2)$

where, v_1 = final velocity of the first body u_1 = initial velocity of the first body, v_2 = final velocity of the second body, u_2 = initial velocity of the second body and

e = constant of proportionality.

Coefficient of Restitution:

The relative velocities of two bodies A and B before and after the impact reflect the loss of kinetic energy. The coefficient of restitution is a measure of the elasticity of impact,

$$\boxed{e = \frac{v_2 - v_1}{u_1 - u_2}}$$

where v_2, v_1 and u_2, u_1 are the velocities of the bodies A and B after and before the impact respectively and e is called the coefficient of restitution.

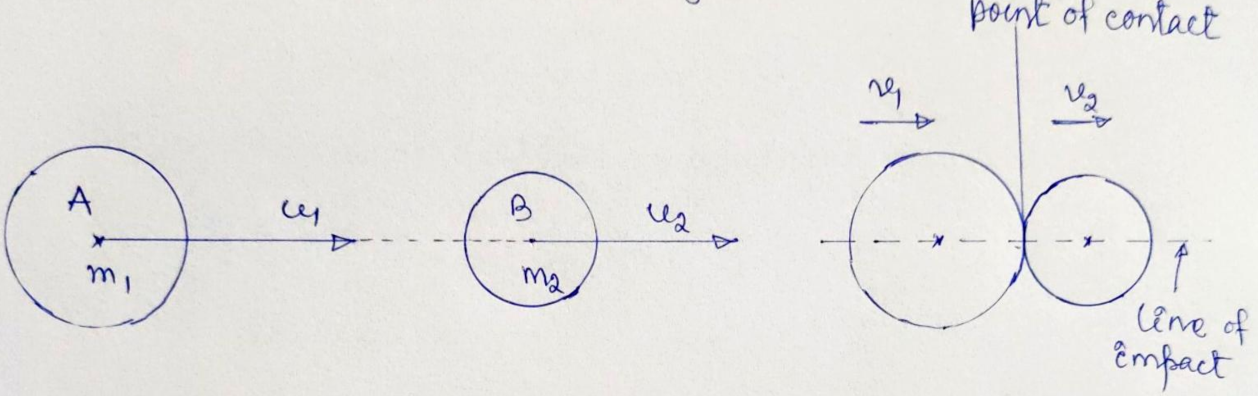
Note

* If the two bodies are moving in the same direction, before and after the impact, then the velocity of approach or separation is the difference of their velocities.
* If they are moving in opposite direction, then the velocity of approach or separation is the algebric sum of their velocities.

* for a perfectly elastic impact, $\boxed{e = 1}$. Both momentum and kinetic energy are conserved.
* for a perfectly plastic impact, $\boxed{e = 0}$. Kinetic energy is not conserved but momentum is conserved.
* for most colliding bodies, the value for e will be between 1 and 0.

Direct Impact of two Bodies :

The collision between two bodies which are moving along the line of impact before the collision is called the direct impact as shown in below fig.



Let m_1 = mass of the first body, u_1 = initial velocity of first body and v_1 = final velocity of the first body.

Similarly, m_2, u_2, v_2 = mass, initial velocity and final velocity of the second body respectively.

The momentum of the body A before collision $= m_1 u_1$

Similarly, for body B $= m_2 u_2$

Hence, the total initial momentum before collision

$$= m_1 u_1 + m_2 u_2$$

On the same line, the total final momentum after collision

$$= m_1 v_1 + m_2 v_2$$

According to the law of conservation of momentum

Total initial momentum = Total final momentum

$$\boxed{m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2}$$

Case - I : Impact of two equal masses :

$$\boxed{m_1 = m_2 = m}$$

then, $m u_1 + m u_2 = m v_1 + m v_2$

or $\boxed{u_1 + u_2 = v_1 + v_2}$ — (i)

The coefficient of restitution relations gives

$$e = 1 = \frac{v_2 - v_1}{u_1 - u_2} \Leftrightarrow \cancel{v_2} -$$

$\Rightarrow u_1 - u_2 = v_2 - v_1$ — (ii)

solving eq^n (i) and (ii), we get

$\boxed{u_1 = v_2}$ and $\boxed{u_2 = v_1}$

After an elastic impact the two masses exchange velocities.

Case- II : Impact of a body on a fixed plane :

Here the fixed plane remains at the same position before and after the impact. Therefore, the initial and the final velocities of a fixed plane are zero.

Let u = initial velocity of the body, v = final velocity of the body and e = coefficient of restitution.

The velocity of approach $= u_1 - u_2 = u - 0 = u$

The velocity of separation $= v_2 - v_1 = 0 - v = -v$

then, $e = \frac{-v}{u}$ or $\boxed{v = -eu}$

Hence, the body strikes the fixed plane with velocity $= v$ and it rebounds with a velocity $= eu$.

Height of Rebound after impact:

Let a body fall freely from a point h m above the fixed plane. The body strikes the plane with a velocity of v is

$$v^2 = u^2 + 2gh$$

$$\Rightarrow v^2 = 2gh \qquad [\because u = 0]$$

$$\Rightarrow v = \sqrt{2gh}$$

After impact, the body will move up with a velocity $= e\sqrt{2gh}$

Let us assume the height of rebound as h^* at which the velocity of the body is zero.

Therefore, $0 = (e\sqrt{2gh})^2 - 2gh^*$

or, $\boxed{h^* = e^2 h}$

Q. A glass marble drops from a height of 3.5 m upon a horizontal floor. If the coefficient of restitution is 0.8, find the height to which it rises after the impact.

Soln: Given data: height of fall, $h =$ ~~0.8~~ 3.5 m, $e = 0.8$

initial velocity of the marble, $u = 0$.

Let v = velocity of the marble before touching the floor

v^* = velocity of the marble after impact

h^* = height to which the marble will rise.

The velocity of marble just before touching the floor

$$v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 3.5} = 8.29 \text{ m/s}$$

The velocity after impact, $v^* = ev = 0.8 \times 8.29 = 6.63$ m/s

Therefore, the height to which the marble will rise,

$$h^* = \frac{v^2}{2g} = \frac{6.63^2}{2 \times 9.81} = \underline{2.24 \text{ m}} \quad \underline{\text{Ans.}}$$

Q. A ball of mass 600 gm is dropped on a horizontal floor from a height of 10 m. The ball rebounds due to impact with the horizontal floor to the height of 6 m. find the coefficient of restitution between the floor and the ball.

Soln: Given data: mass of the ball = 600 gm = 0.6 kg,

$u = 0$; $h = 10$ m and $h^* = 6$ m.

Let v = velocity with which the ball will impact the floor

Applying the equation, $v^2 = u^2 + 2gh$

$$v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 10} = 14 \text{ m/s}$$

The above velocity is the velocity of approach.

Now consider u = velocity of separation.

$$v^2 = u^2 - 2gh^*$$

$$0 = u^2 - 2 \times 9.81 \times 6$$

$$\Rightarrow u = \sqrt{2 \times 9.81 \times 6} = 10.84 \text{ m/s}$$

$$e = \frac{u}{v} = \frac{10.84}{14} = \underline{0.77} \quad \underline{\text{Ans.}}$$

Q. A ball A of mass 2 kg moving with a velocity of 1.5 m/s strikes directly on a ball B of mass 4 kg at rest. The ball A after striking the ball B comes to rest. find the velocity of the ball B after striking and the coefficient of restitution.

Soln: $m_a = 2$ kg, $u_a = 1.5$ m/s, $m_b = 4$ kg $u_b = 0$, $v_a = 0$

v_b = final velocity of ball B.

According to the law of conservation of momentum,

$$m_a u_a + m_b u_b = m_a v_b + m_b v_b$$

$$\Rightarrow 2 \times 1.5 + 4 \times 0 = 2 \times 0 + 4 \times v_b$$

or $v_b = \frac{3}{4} = 0.75 \text{ m/s}$ Ans.

$$e = \frac{v_b - v_a}{u_a - u_b} = \frac{0.75 - 0}{1.5 - 0} = \frac{1}{2} = 0.5$$ Ans.

Q. A ball of mass 30 kg moving with a velocity of 4 m/s strikes directly another ball of mass 15 kg moving in opposite direction with a velocity of 6 m/s. If the coefficient of restitution is 0.8, determine the velocity of each ball after impact.

Solⁿ: mass of first ball, $m_1 = 30$ kg, initial velocity of m_1, $u_1 = 4$ m/s, $m_2 = 15$ kg, $u_2 = -6$ m/s, $e = 0.8$.

Let v_1 & v_2 = velocity of ~~mass~~ first and second ball after impact.

According to law of conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Rightarrow 30 \times 4 + 15 \times (-6) = 30v_1 + 15v_2$$

$$\Rightarrow 30v_1 + 15v_2 = 120 - 90 = 30$$

$$\Rightarrow 2v_1 + v_2 = 2 \quad \text{--- (1)}$$

& $$e = \frac{v_2 - v_1}{u_1 - u_2} \Leftrightarrow 0.8 = \frac{v_2 - v_1}{4 - (-6)}$$

$$\Leftrightarrow v_2 - v_1 = 10 \times 0.8 = 8$$

$$\Rightarrow v_2 = v_1 + 8 \quad \text{--- (2)}$$

Substituting v_2 in eqⁿ (1), we get

$$2v_1 + v_1 + 8 = 2 \Leftrightarrow \cancel{16v_1 =}$$

$$\Rightarrow 3v_1 = 2 - 8 = -6$$

$$\Rightarrow v_1 = -\frac{6}{3} = -2 \text{ m/s}$$ Ans.

$$v_2 = v_1 + 8 = -2 + 8 = 6 \text{ m/s}$$ Ans.