FRICTION

The force of friction or simply friction is the force distribution at the surface of contact between the two bodies that prevents or impedes the sliding motion of one body relative to the other. This force distribution is tangent to the contact surface and has for the body under consideration, a direction at every point in the contact surface that is in opposition to the possible or existing slipping motion of the body at that point.

Dry friction:

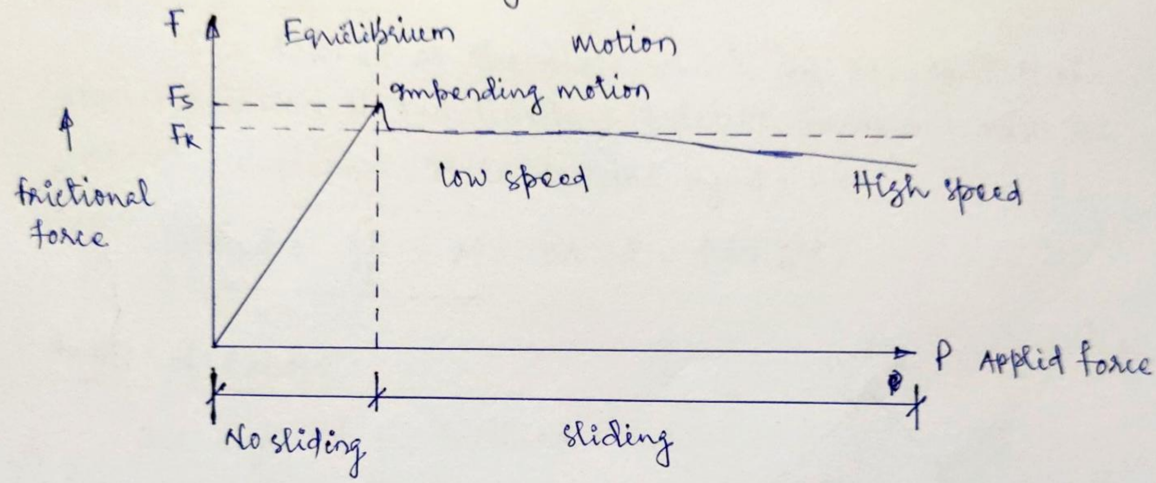
Coulomb or dry friction is that friction which occurs between bodies having dry contact surfaces. The major cause for dry friction is believed to be the microscopic roughness of the surfaces of contact. Interlocking of microscopic protuberance oppose the relative motion between the surfaces. When sliding is present between the surfaces, some of these protuberances either is sheared off or is melted by high local temperatures.

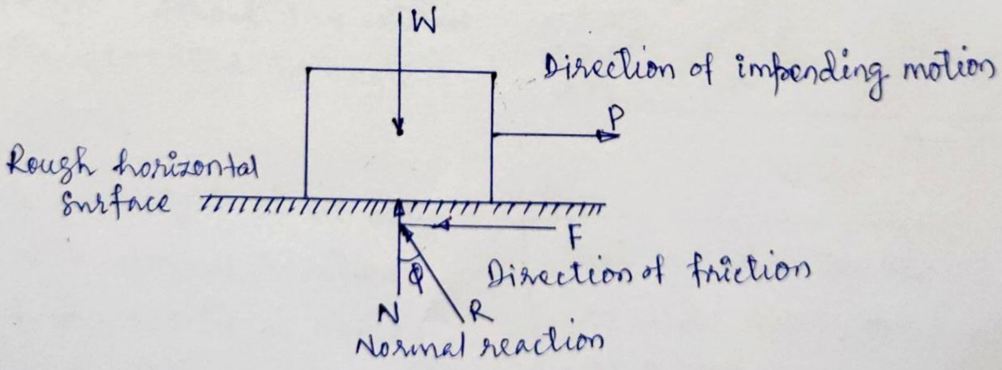
A smooth surface can only support a normal force, whereas a rough surface in addition to normal force can support a force tangent to the contact surface (i.e. the friction force).

Limiting friction:

Let us assume a block of weight W is resting on a flat surface. A small horizontal force P is applied to the block and is increased slowly. Equilibrium exists only if the frictional force F equals the applied force P.

So, F also increases slowly, until sliding is impending. If P is just large enough for sliding to be impending slightly the block slides, then F decreases to its kinetic value F_k, hence, equilibrium can no longer be maintained unless P is reduced accordingly. Depending on the material properties of the bodies, the magnitude of the frictional force F may drop F_k at high speed of sliding.





Coefficient of friction

It is defined as the ratio of limiting force of friction to the ~~no~~ normal reaction between the two bodies. It is denoted by μ (mue).

$$\boxed{\mu = \frac{F}{N}} \quad \text{or} \quad \boxed{\mu = \tan\phi}$$

Coefficient of static friction,

$$\boxed{\mu_s = \frac{F_s}{N}}$$

Coefficient of kinetic friction,

$$\boxed{\mu_k = \frac{F_k}{N}}$$

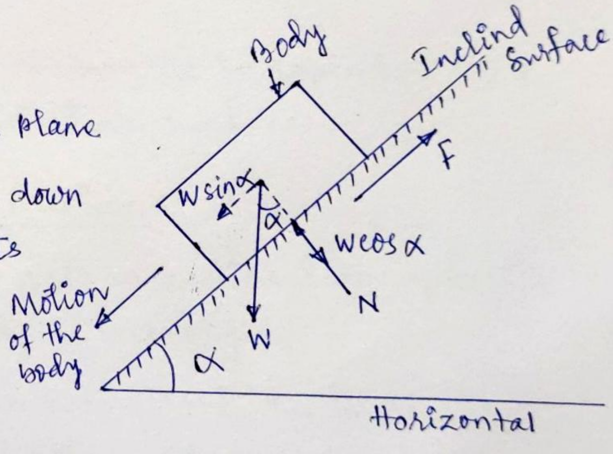
Angle of friction :

It is defined as the angle which the resultant R of normal reaction N and limiting friction F_s makes with the normal reaction. It is denoted by ϕ, i.e.

$$\boxed{\tan\phi = \frac{F_s}{N} = \mu_s} \quad \text{or} \quad \phi = \tan^{-1}(\mu_s)$$

Angle of Repose :

The angle of the inclined plane where the body is about to slide down the inclined plane mainly by its weight without any external force is called the angle of repose.



W → Weight of the body

N → Normal reaction

f → frictional force.

$\Sigma F_H = 0$; $F = W\sin\alpha$

$\Sigma F_V = 0$; $N = W\cos\alpha$

Then $\frac{F}{N} = \frac{W\sin\alpha}{W\cos\alpha} = \tan\alpha$

But $\frac{F}{N} = \mu = \tan\phi$

or $\tan\alpha = \tan\phi$

or $\boxed{\alpha = \phi}$

That is, when the body is about to slide down, the inclination of the plane to the horizontal is equal to the angle of friction.

$$\boxed{\alpha = \phi}$$

α → angle of repose

ϕ → angle of friction.

Laws of Friction

Laws of static friction:

1. when two surfaces are in contact, the frictional force always acts tangential to the contacting surfaces in a direction opposite to that in which the body tends to move.
2. The frictional force is, with in the limits, proportional to the normal reaction force existing between the surfaces in contact, i.e.

$$\boxed{F_s = \mu_s N}$$

3. The frictional force depends upon the nature of the surface in contact.
4. The coefficient of static friction is independent of the area and shape of the contacting surfaces.

Laws of Dynamic or kinetic friction:

1. The force of friction always acts in a direction opposite to that in which the ~~m~~ body is moving.
2. The frictional force remains constant for moderate speed but it decreases slightly with the increase of speed.
3. The magnitude of the kinetic friction has a constant ratio to the normal reaction between the two surfaces. But this ratio is slightly less than that in case of limiting friction.

$$\boxed{F_K = \mu_K N}$$

4. The coefficient of the kinetic friction is independent of the area and shape of the contacting surfaces.

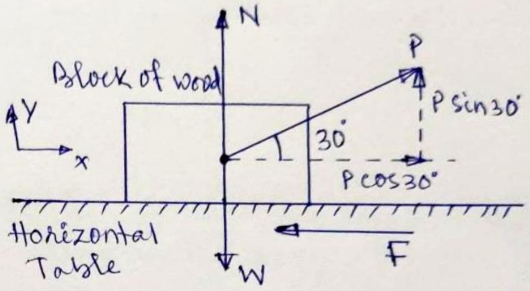
Equilibrium of Body on Rough Horizontal Plane

Q. A pull of 25 kN at 30° to the horizontal is necessary to move a block of wood on a horizontal table. If the coefficient of friction between the bodies in contact is 0.2, what is the weight of the block?

Sol^n: Given data:

Pull $P = 25\ kN$, $\theta = 30^\circ$, $\mu = 0.2$

W = unknown weight of the block



Under equilibrium conditions,

$\Sigma F_x = 0;\quad F = P\cos 30^\circ$

or $\mu N = P\cos 30^\circ$ — ①

$\Sigma F_y = 0;\quad N + P\sin 30^\circ = W$

or $N = W - P\sin 30^\circ$ — ②

From eq^n ① and ②, we get

$\mu(W - P\sin 30^\circ) = P\cos 30^\circ$

or $\mu W - \mu P\sin 30^\circ = P\cos 30^\circ$

or $W = \dfrac{P\cos 30^\circ + \mu P\sin 30^\circ}{\mu} = \dfrac{25\cos 30^\circ + 0.2\times 25\sin 30^\circ}{0.2}$

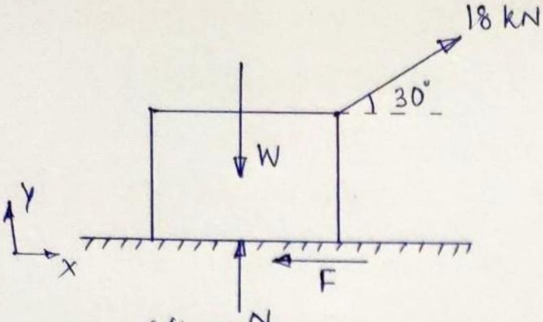
or $W = 120.75\ kN$ Ans.

Q. A body resting on a rough horizontal plane required to pull 18 kN inclined at 30° to the plane just to remove it. It was found that a push of 22 kN inclined at 30° to the plane just removed the body. Determine the weight of the body and the coefficient of friction.

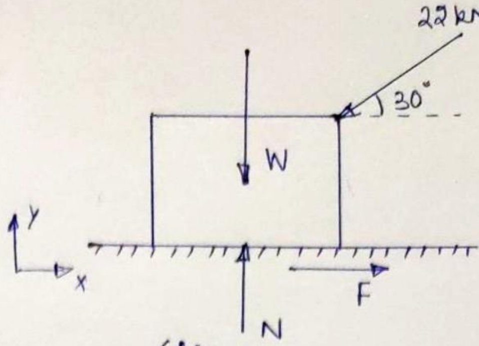
Sol^n: Given data: Pull = 18 kN, Push = 22 kN, $\theta = 30^\circ$

W = Weight of the body, N = Normal reaction

μ = coefficient of friction.



(fig.1)



(fig.2)

Case-I: considering the pull acting on the body (fig.1).

$\Sigma F_x = 0$; $F = 18\cos 30°$

or, $F = 18 \times 0.866 = 15.6$ kN

$\Sigma F_y = 0$; $N = W - 18\sin 30°$

or, $N = W - 18 \times 0.5 = (W-9)$ kN

$F = \mu N$

$\Rightarrow 15.6 = \mu(W-9)$ — ①

Case-II: considering the push acting on the body (fig.2).

$\Sigma F_x = 0$; $F = 22\cos 30°$

or, $F = 22 \times 0.866 = 19.05$ kN

$\Sigma F_y = 0$; $N = W + 22\sin 30°$

or, $N = W + 22 \times 0.5 = (W+11)$ kN

$F = \mu N$

$\Rightarrow 19.05 = \mu(W+11)$ — ②

Dividing eq^n ① by eq^n ②, we get

$$\frac{15.6}{19.05} = \frac{\mu(W-9)}{\mu(W+11)} = \frac{W-9}{W+11}$$

or $15.6(W+11) = 19.05(W-9)$

or $15.6W - 19.05W = -19.05 \times 9 - 15.6 \times 11$

or $-3.45W = -171.45 - 171.6 = -343.05$

or $W = \frac{-343.05}{-3.45} = \underline{99.4 \text{ kN}}$ Ans.

Thus, substituting the value of W in either eq^n ① or eq^n ②, we get

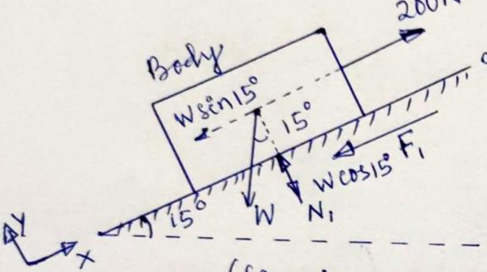
$15.6 = \mu(99.4 - 9) = 90.4\mu$

$\Rightarrow \mu = \frac{15.6}{90.4} = \underline{0.173}$ Ans.

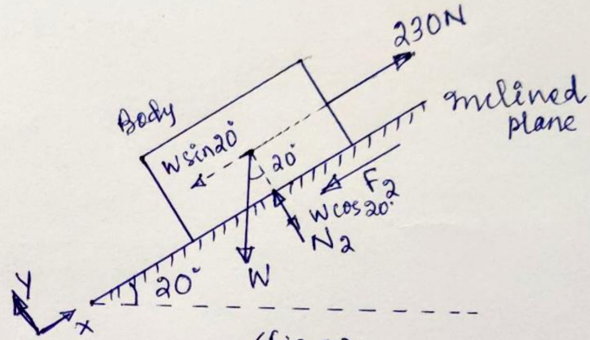
Equilibrium of Body on Rough Inclined Plane

Q. An effort of 200 N is required just to move a certain body up an inclined plane of an angle 15°, the force acting parallel to the plane. If the angle of inclination of the plane is made 20°, the effort required again applied parallel to the plane is found to be 230N. find the weight of the body and the coefficient of friction.

Sol^n:



(fig. 1)



(fig. 2)

Case - I

Consider a body which is lying on an inclined plane making an angle 15° with the horizontal (fig. 1).

Under equilibrium condition,

$\Sigma F_x = 0;$ $\quad 200 = F_1 + W \cdot \sin 15°$

$$200 = \mu N_1 + W \sin 15° \quad — (1)$$

$\Sigma F_y = 0;$ $\quad N_1 = W \cos 15°$

Substituting the value of N_1 in eq^n (1), we get

$$200 = \mu (W \cos 15°) + W \sin 15°$$

$$200 = W(\mu \cos 15° + \sin 15°) \quad — (2)$$

Case - II : consider the body lying on an inclined plane which is inclined at an angle of 20° with the horizontal (fig. 2).

$\Sigma F_x = 0;$ $\quad 230 = F_2 + W \sin 20°$

or $\quad 230 = \mu N_2 + W \sin 20°$ — (3)

$\Sigma F_y = 0;$ $\quad N_2 = W \cos 20°$

$\therefore \quad 230 = \mu (W \cos 20°) + W \sin 20°$

$230 = W(\mu \cos 20° + \sin 20°)$ — (4)

Divide eqⁿ (4) by eqⁿ (2), we get

$$\frac{230}{200} = \frac{W(\mu \cos 20° + \sin 20°)}{W(\mu \cos 15° + \sin 15°)}$$

Upon simplification, we get

$\mu = \underline{0.259}$ Ans.

Thus, $\quad 230 = W(\mu \cos 20° + \sin 20°)$

$$W = \frac{230}{\mu \cos 20° + \sin 20°} = \frac{230}{0.259 \times 0.939 + 0.342}$$

$W = \underline{393.03 \text{ N}}$ Ans.

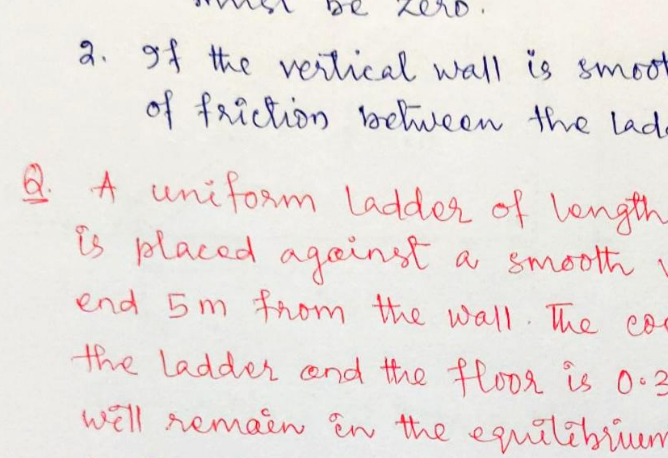
Ladder Friction

The following points must be considered in the analysis of ladder problem:

1. Apply the conditions of equilibrium, such as:
   (a) The algebric sum of the horizontal and vertical components of the forces must be zero, and,
   (b) The moments of all the forces about any point must be zero.
2. If the vertical wall is smooth, there will be no force of friction between the ladder and the vertical wall.

Q. A uniform ladder of length 13 m and weighing 25 kN is placed against a smooth vertical wall with its lower end 5 m from the wall. The coefficient of friction between the ladder and the floor is 0·3. Show that the ladder will remain in the equilibrium in this position. What is the frictional force acting on the ladder at the point of contact between the ladder and the floor?

Sol^n:



Since the ladder is placed against a smooth vertical wall, the frictional force will be zero at the point of contact between the ladder and the wall.

$\Sigma F_x = 0; \quad F = N_C$

$\Sigma F_y = 0; \quad N_A = 25$

$BC = \sqrt{AC^2 - AB^2} = \sqrt{13^2 - 5^2} = 12\,m$

$\Sigma M_C = 0; \quad F \times 12 - N_A \times 5 + 25 \times 2{\cdot}5 = 0 \Leftrightarrow F = \dfrac{25 \times 5 - 25 \times 2{\cdot}5}{12} = 5{\cdot}21\text{ kN}$

But, the maximum frictional force developed at the floor is

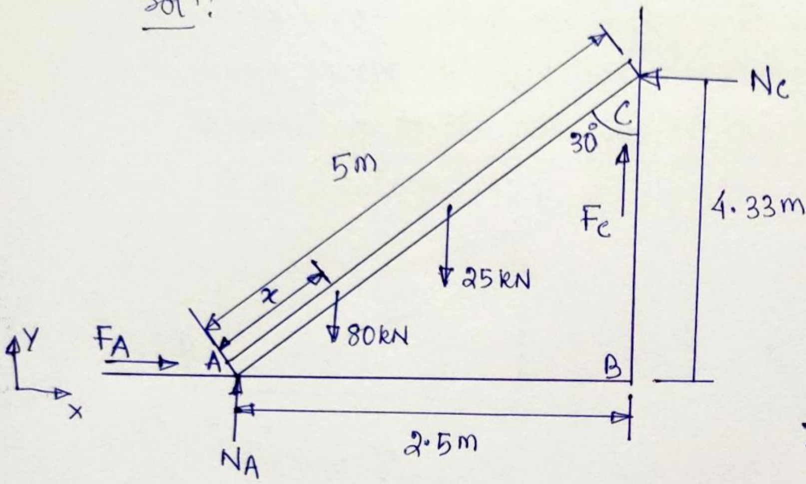
$F = \mu N_A = 0{\cdot}3 \times 25 = 7{\cdot}5\text{ kN}$

$\boxed{5{\cdot}21 < 7{\cdot}5}$

Hence, the ladder will remain in equilibrium.

Q. A ladder 5 m long and 25 kN weight is placed against a vertical wall in a position where its inclination to the vertical is 30°. A man weighing 80 kN climbs the ladder. At what position will ~~be~~ he induce slipping? The coefficient of friction for all the contact surface is 0.2.

Soln:



Consider x is the distance from the foot of the ladder where the man feels slipping of the ladder.

Under equilibrium condition,

$\Sigma F_x = 0;\ F_A = N_C$

$F_A = \mu N_A = 0.2 N_A = N_C \quad — (1)$

$\Sigma F_y = 0;\quad N_A = 80 + 25 - F_C = 105 - F_C$

substituting the value of N_A in eqn (1), we get

$0.2(105 - F_C) = N_C$

$\left[\because F_C = \mu N_C = 0.2 N_C\right]$

$\Rightarrow 0.2(105 - 0.2 N_C) = N_C$

$\Rightarrow 0.2 \times 105 - 0.2 \times 0.2 N_C = N_C$

$\Rightarrow N_C = \underline{20.19\ kN}\ ;\quad F_C = 0.2 \times 20.19 = \underline{4.038\ kN}$

$F_A = N_C = \underline{20.19\ kN}$

$\Sigma M_A = 0;\quad 80 x \sin 30^\circ + 25 \times 2.5 \sin 30^\circ = N_C \times 4.33 + F_C \times 2.5$

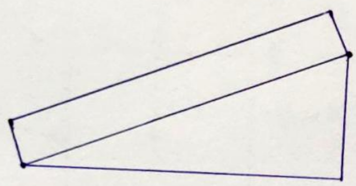
$$x = \frac{20.19 \times 4.33 + 4.038 \times 2.5 - 25 \times 2.5 \sin 30^\circ}{80 \sin 30^\circ}$$

$x = \underline{\underline{1.656\ m}}$ Ans.

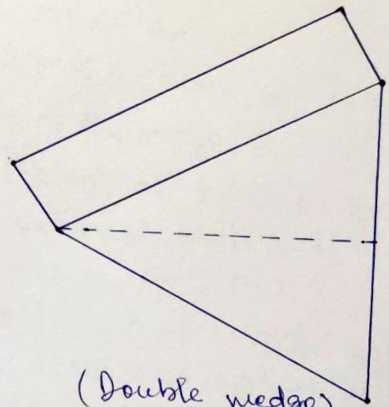
Wedge Friction :

Wedges are among the oldest and simplest machines. They are still very useful in amplifying the effects of small forces or making precise small adjustments in the positions of bodies.

Friction plays an important role in the behaviour of every wedge. Wedges are generally made of wood or metal according to its use and of triangular or trapezoidal in cross-section.



(Single wedge)

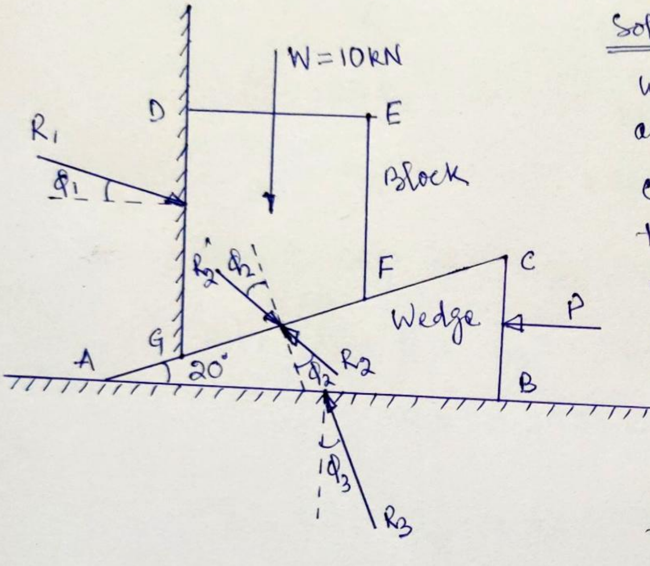


(Double wedge)

They are used as lifting devices, as keys or cotter for shafts, tightening fits, etc. They have flat horizontal bottom surface and inclined plane surface at the top in the case of single wedge and both surfaces inclined at equal angles to the central plane in the case of double wedge.

The problems on wedge are generally the problems of equilibrium on inclined planes. Therefore, these problems are solved by equilibrium method or by applying Lami's theorem.

Q. A block weighing 10kN is to be raised with the help of a wedge of 20° as shown in fig. If the coefficient of friction between the vertical wall and the surface of block is 0.2, while that between the floor and the surface of the block is 0.3. Determine the magnitude and direction of the minimum horizontal force required to lift the block. Assume the coefficient of friction between the block and wedge surface as 0.25.



Sol^n: Given data:

Weight of the block, W = 10 kN

angle of the wedge, $\alpha = 20°$

coefficient of friction between the vertical wall and the surface of block, $\mu_1 = 0.2$

coefficient of friction between the floor and surface of the block, $\mu_2 = 0.3$.

coefficient of friction between the block and the wedge surface $\mu_3 = 0.25$.

Therefore $\phi_1 = \tan^{-1}(\mu_1) = \tan^{-1}(0.2) = 11.31°$

$\phi_2 = \tan^{-1}(\mu_2) = \tan^{-1}(0.3) = 16.70°$

$\phi_3 = \tan^{-1}(\mu_3) = \tan^{-1}(0.25) = 14.04°$

Sliding takes place along AB, AC and GD. The reaction of the wall R_1 on the block will be inclined at an angle ϕ_1 with the normal to the rubbing surface GD. Similarly, the reaction R_2 of the wedge surface AC on the block surface will be inclined at an angle ϕ_2 with the normal to the rubbing surface AC. On the same way, the reaction of the block surface on the surface of the wedge will be R_2 equal to R_2 inclined at ϕ_2 with the normal to surface AC.

The reaction R_3 of the floor on the wedge surface AB will be inclined at ϕ_3 with the normal to surface AB.

Under equilibrium conditions of the block,

$\Sigma F_x = 0$; $R_1 \cos 11.31° = R_2 \sin(20° + 16.70°)$

$0.9805 R_1 = R_2 \sin(36.7°)$

$R_1 = 0.609 R_2$

$\Sigma F_y = 0$; $R_1 \sin 11.31° + 10 = R_2 \cos 36.7°$

$(0.609 R_2) \sin 11.31° + 10 = R_2 \cos 36.7°$

$R_2 = 14.65$ kN

Now, considering the equilibrium of the wedge

$\Sigma F_x = 0$; $P = R_3 \sin 14.04° + R_2' \sin 36.7°$

$P = 0.2425 R_3 + 0.5976 R_2'$

$\Sigma F_y = 0$; $R_2' \cos 36.7° = R_3 \cos 14.04°$

$R_3 = 12.11$ kN

Thus $P = \underline{11.69 \text{ kN}}$ Ans.