Engineering Mechanics

Mechanics:

Mechanics may be defined as the branch of science which deals with the state of rest or motion of bodies under the action of forces.

Mechanics is divided into two parts:

1. Statics:- It deals with the equilibrium of bodies under the action of forces
2. Dynamics:- Concerns the motion of bodies.

Rigid Body:

A body is considered rigid when the change in distance between any two of its points is fixed, both before and after applying a load. In most cases the actual deformations occurring in structures, machines, etc. are relatively small, and the rigid-body assumption is suitable for analysis.

Force:

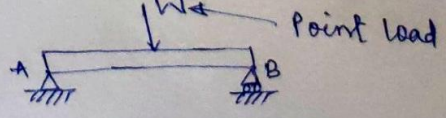
force can be defined as an agency, which changes or tends to change the state of conditions (i.e. rest or motion) of a body.

In order to define a force in details we must specify its magnitude, direction and point of application.

The magnitude of a force is expressed in newton (N)

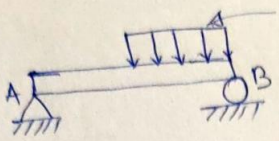
Concentrated force/Point load:

If the forces are acting on a body assumed to be concentrated at a specific point on that body are reffered to be concentrated or point load.



Distributed forces:

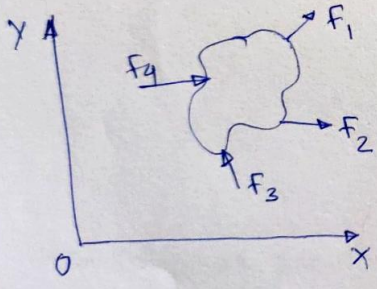
Here the forces are acting over a finite areas of the surface of the body, are termed as distributed forces.



force system

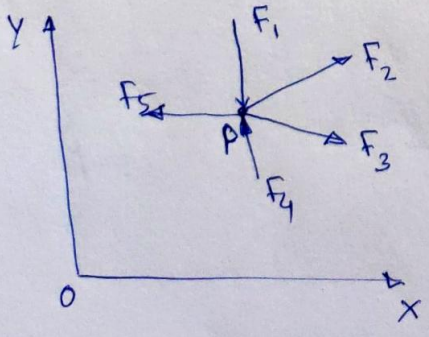
1. Coplanar force systems
   - Concurrent force system
   - Parallel force system
   - Non-concurrent force system
2. Non-coplanar force systems
   - Concurrent force system
   - Parallel force system
   - Non-concurrent force system

Coplanar force system



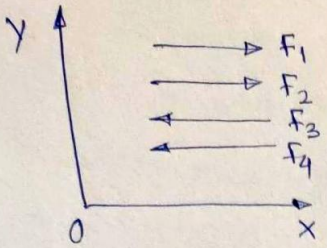
Lines of action of the forces F_1, F_2, F_3 and F_4 are lie on one plane (x-y)

Coplanar concurrent force system:



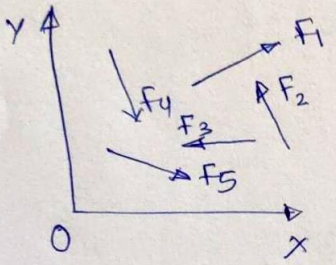
Lines of action of forces F_1, F_2, F_3, F_4 and F_5 are meeting at a common point 'P'.

Coplanar Parallel force system



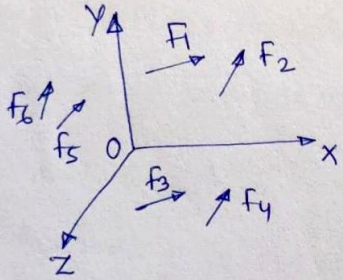
Line of action of the forces are parallel to one another.

~~Non~~ Coplanar Non-Concurrent force system

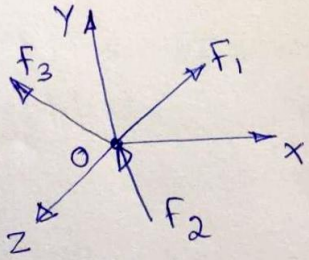


line of action of forces are different and do not meet at a common point.

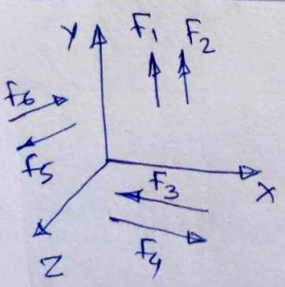
Non-Coplanar force system



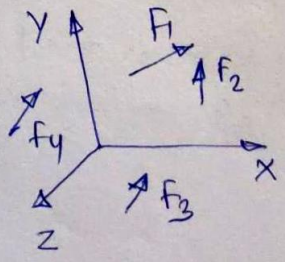
Non-coplanar Concurrent force



Non-Coplanar parallel force



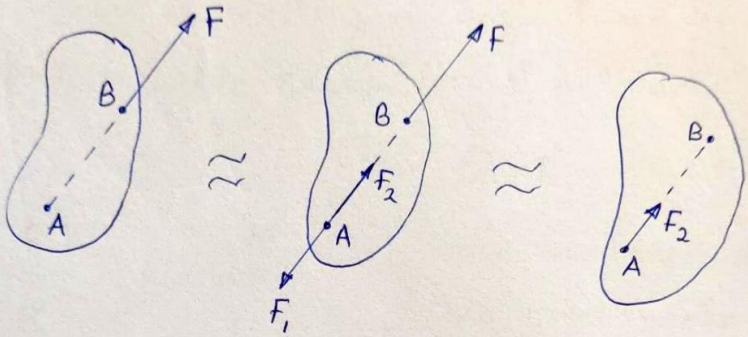
Non-Coplanar Non-Concurrent force



Principle of Superposition

The action of a given system of force on a rigid body will in no way be changed if we add to or substracted from them another system of forces in equilibrium.

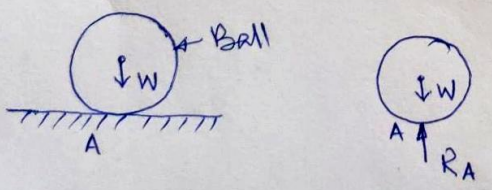
Principle of Transmissibility



The point of application of a force may be transmitted along its lines of action without altering the net effect on the rigid body to which it may be applied.

Action and Reaction:

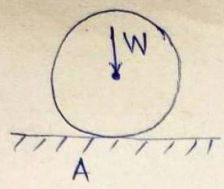
According to Newton's 3rd laws of motion, 'to every action there is an equal and opposite reaction'.

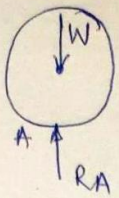


self weight of the ball, W is action upward force exerted on the ball from the surface, R_A is reaction.

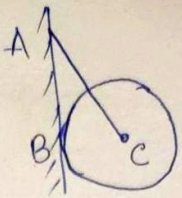
Free Body Diagram (FBD)

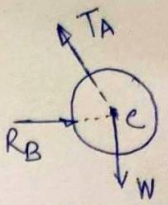
To analyse the equilibrium of a constrained body, we remove the supports and replace them by their reactions.





(FBD)

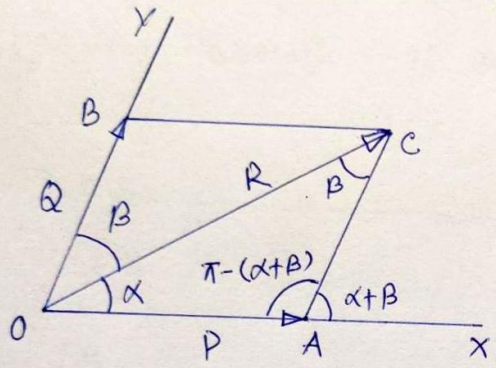




(FBD)

Resolution of a force

The process of spliting a single force by several components is caled resolution of a force.



$\bar{OA} = \vec{P}$, $\bar{OB} = \vec{Q}$, $\bar{OC} = \vec{R}$

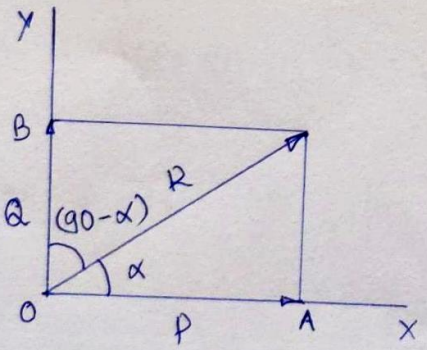
Here the forces P and Q are two components of the single force R as follows.

$$P = R \frac{\sin\beta}{\sin(\alpha+\beta)}$$

$$Q = R \frac{\sin\alpha}{\sin(\alpha+\beta)}$$

Note If $\boxed{\alpha+\beta = 90^\circ}$, then $P = R \dfrac{\sin(90^\circ-\alpha)}{\sin 90^\circ} = R\cos\alpha$

$$Q = R \frac{\sin\alpha}{\sin 90^\circ} = R\sin\alpha$$



$\boxed{P = R\cos\alpha}$

$\boxed{Q = R\sin\alpha}$

} Rectangular component of force R.

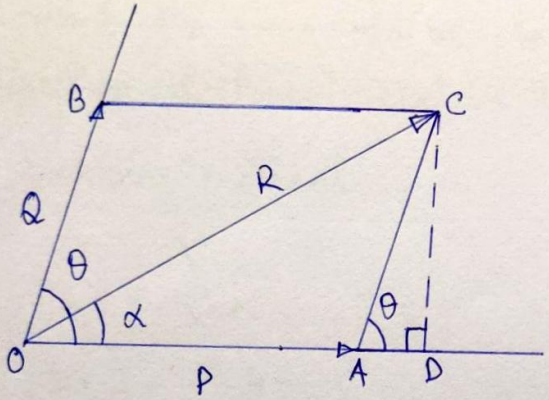
* Care must be taken in the measurement of angle between the line of actions of the given forces and their directions.

Composition of forces

Addition of forces is called composition of forces.

Parallelogram law

If two forces, which act at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from one of its angular point, their resultant is represented by the diagonal of the parallelogram passing through that angular point, in magnitude and direction.



Resultant, R

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$$

Case-I $\boxed{\theta = 0^\circ}$; $\boxed{R = P + Q}$

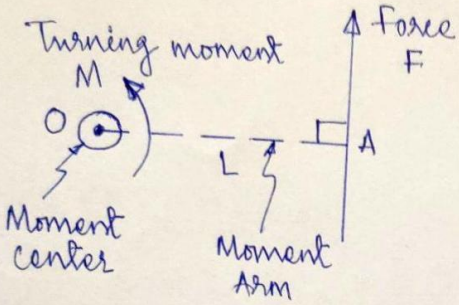
Case-II $\boxed{\theta = 90^\circ}$; $\boxed{R = \sqrt{P^2 + Q^2}}$

Case-III $\boxed{\theta = 180^\circ}$; $\boxed{R = P - Q}$ if $\boxed{P > Q}$

Case-IV $\boxed{P = Q}$; $\boxed{R = 2P\cos\frac{\theta}{2}}$ and $\boxed{\alpha = \frac{\theta}{2}}$

Moment of a force

A force can produce rotation about a certain fixed point on a body to which it is applied is known as moment of a force.



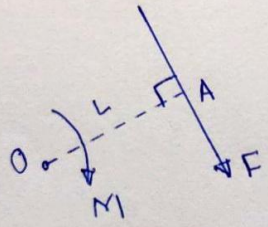
Moment = moment arm x magnitude of applied force

$$M = L \times F$$

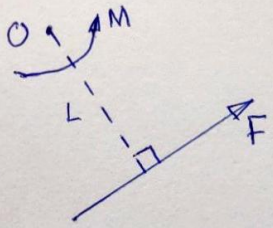
Unit : in SI - N.m

Moment of a force can be classified according to the direction of force relative to moment center.

clockwise moment



Anti-clockwise moment



Note.

The moment of a force can be equal to zero under two conditions. i.e.

i) Either the applied force is zero.

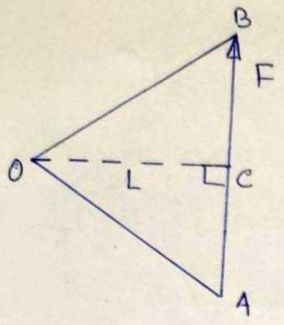
ii) The moment center lies on the line of action of the applied force.

Equilibrium

If a body is in equilibrium under a system of coplanar non-concurrent force, the algebric sum of all moments about any point on a body is equal to zero. i.e.

$$\Sigma M_o = 0$$

Geometrical meaning of moment of a force



Moment of a force is equal to twice the area of the triangle whose vertex is the moment center and base represents the applied force.

$M = 2 \times$ Area of triangle OAB

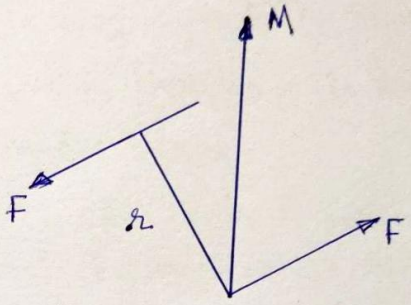
Varignon's theorem

"If a body is acted upon by a number of coplanar forces, then the algebraic sum of the moments of all the forces about any point in the body is equal to the moment of their resultant force about the same point."

Couple (C)

A force couple or simply a couple is defined as a pair of equal, parallel, oppositely directed forces. The perpendicular distance r between the lines of actions of the forces is called the moment arm of the couple. The vector sum of these two forces is zero, but their moment sum is not equal to zero.

The only effect of a couple on a body is a tendency to rotate the body about an axis perpendicular to the ~~axis~~ plane of the couple.



Vector form:

The moment of the couple is

$$M_c = r \times F = |r||F| \sin 90^\circ = |r||F|$$

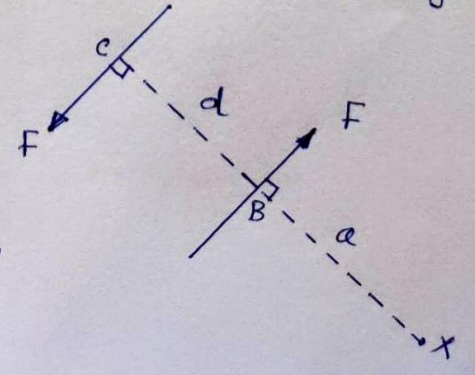
where F = parallel forces of equal amount

r = moment arm of the couple

This moment is a vector quantity and acts along the normal to the plane containing F and r guided by the right-hand rule.

Scalar form:

A unique property of a couple is that the moment sum of its forces is constant and independent of any moment center.



$$\Sigma M_B = Fd$$

$$\Sigma M_x = F(d+a) - Fa = Fd$$

Therefore, one can conclude as, the moment of a couple (M_c) is

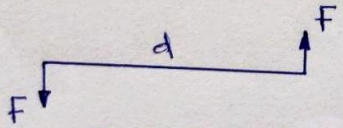
equal to the product of one of the forces composing the couple multiplied by the perpendicular distance (d) between their action lines.

Since the only effect of a couple is to produce a moment that is independent of the moment center, the effect of couple is unchanged if:

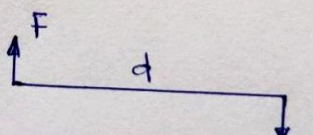
i) the couple is rotated through any angle in its plane.
ii) the couple is shifted to any other position in its plane.
iii) the couple is shifted to a parallel plane.
iv) the couple is replaced by any other pair of forces in its plane whose product fd and sense of rotation is unchanged.

Sense of a Couple:

A couple is not determined solely by its plane and the magnitude of its moment. The sense of couple is also very important.



Anticlockwise



Clockwise

Examples of Couple:

1. Two equal and unlike parallel forces acting on the knob of water tap when opening or closing it.
2. Two equal and unlike parallel forces acting on the handle of a bicycle or bike while taking a turn.

Equilibrium

The concept behind the equilibrium of a body in engineering mechanics, mainly refers to the state of the body wheather it is at rest or in motion. Therefore, if the state of a body is at rest is in equilibrium then it is called static equilibrium. On the other hand, dynamic equilibrium means a body is in the state of motion with uniform velocity.

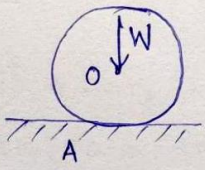
Equilibrium of a body is classified as follows:

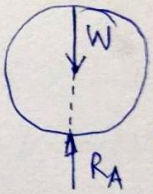
i) Two forces system
ii) Three forces system
iii) General forces system

Two forces system (

Conditions for equilibrium of two forces are as follows:

i) equal in magnitude
ii) opposite in direction
iii) Colinear in action

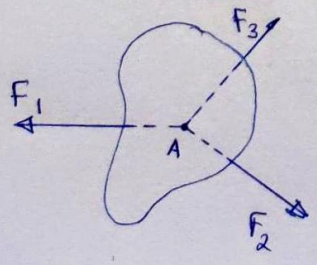


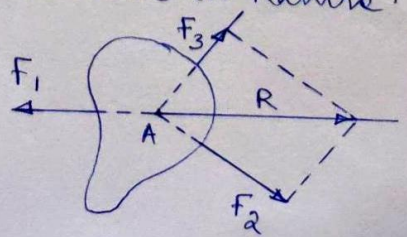


$R_A = W$

Three forces system

Three non-parallel forces attain equilibrium only if their line of action must meet at a common point. It means the requisite condition to be in equilibrium, the three forces must be concurrent in nature.





i) $F_1 = R$
ii) F_1 & R colinear
iii) F_1 & R opposite in direction

General force system :

1. The algebric sum of components of all the forces in the horizontal direction must be zero. i.e.

$$R_x = \Sigma F_x = 0$$

2. The algebric sum of components of all the forces in the vertical direction must be zero. i.e.

$$R_y = \Sigma F_y = 0$$

Thus, the resultant $R = 0$

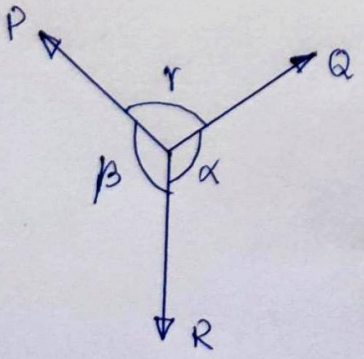
These two conditions must be fulfilled by a system of concurrent forces, if the body on which they act is in equilibrium.

for a system of coplanar, non-concurrent forces;

$$\Sigma F_x = 0 ; \quad \Sigma F_y = 0 ; \quad \Sigma M_c = 0$$

Lami's theorem :

Lami's theorem states that if a body is in equilibrium under the action of three forces, then each force is proportional to the sine of the angle between the other two forces.



$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin r}$$