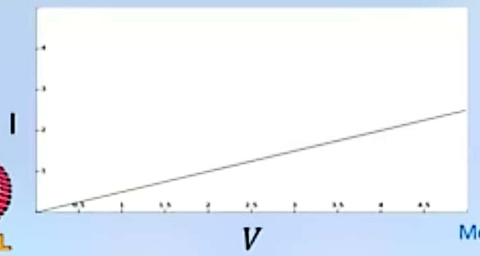
Classification of Systems

- Variety of classifications are possible based on system features and applications
- Some of the important classifications include:
  - Linear and non-linear systems
  - Static and dynamic systems
  - Time invariant and time variant systems
  - Causal and non-causal systems

Linear Vs Non-Linear Systems

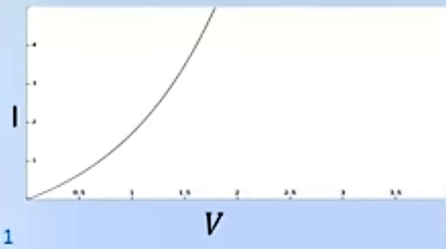
Linear systems

- Output of the system varies linearly with input
- Satisfy homogeneity and superposition
- E.g. Resistor : $I = \frac{V}{R}$



Non-linear systems

- Output of the system does not vary linearly with input
- Do not satisfy homogeneity and superposition
- E.g. Diode: $I = I_0(e^{\frac{V}{\tau}} - 1)$



Static Vs Dynamic Systems

Static systems

- At any time, output of the system depends only on present input
- Memory less systems

- $y(t) = f(u(t))$
- E.g. Resistor:

$$I(t) = \frac{V(t)}{R}$$

Dynamic systems

- Output of the system depends on present as well as past inputs
- Presence of memory can be observed
- $y(t) = f(u(t), u(t-1), u(t-2), \dots)$
- E.g. Inductor:

$$I(t) = \frac{1}{L}\int_0^t V(t)dt$$

Time Invariant Vs Time Variant Systems

Time invariant systems

- Output of the system is independent of the time at which the input is applied
- $y(t) = f\big(u(t)\big) \Rightarrow y(t+\delta) = f(u(t+\delta))$
- E.g. An ideal resistor

$$I(t) = \frac{V(t)}{R} \Rightarrow I(t+\delta)\ \frac{V(t+\delta)}{R}$$

Time variant systems

- Output of the system varies dependent on the time at which input is applied
- $y(t) = f\big(u(t)\big) \nRightarrow y(t+\delta) = f(u(t+\delta))$
- E.g. Aircraft: Mass (M) of aircraft changes as fuel is consumed
- Acceleration: $a(t) = \frac{F(t)}{M(t)}$

Causal Vs Non-causal Systems

Causal systems

- Output is only dependent on inputs already received (present or past)
- Non-anticipatory system
- $y(t) = f(x(t), x(t-1), ..)$
- E.g.
  - Thermostat based AC
  - Motor or generator

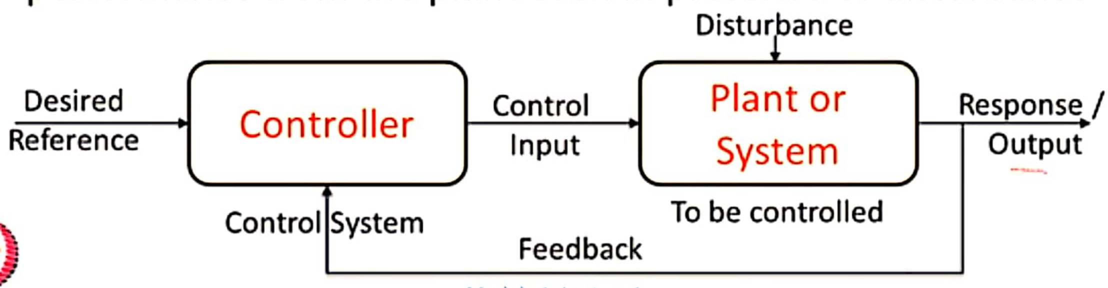
Non-causal systems

- Output depends on future inputs as well
- System anticipates future inputs based on past
- $y(t) = f(x(t), x(t+1), ..)$
- E.g.
  - Weather forecasting system
  - Missile guidance system

NPTEL

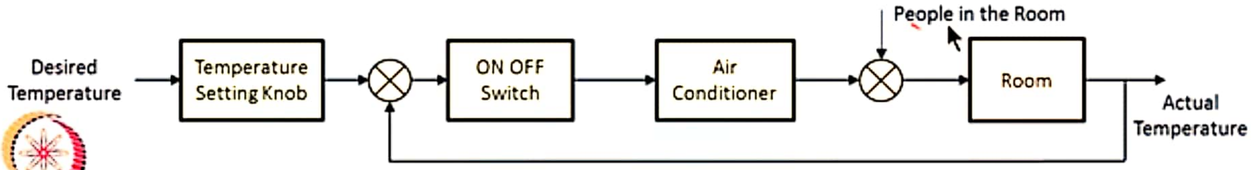
Feedback in Control

- Feedback senses the plant output and gives a signal which can be compared to the reference
- Controller action (control input) changes based on feedback
- Feedback enables the control system in extracting the desired performance from the plant even in presence of disturbance



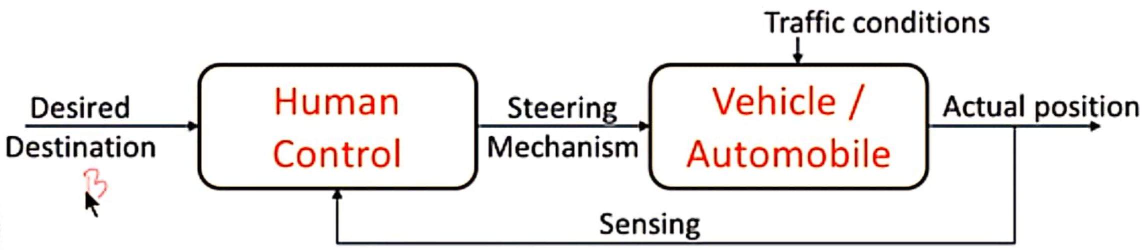
Examples of Control Systems

- **Air conditioner maintaining desired temperature:**
  - Plant : Room
  - Control system : Air Conditioner
  - Reference : Desired temperature
  - Control Input : Compressor ON/OFF
  - Output : Output temperature
  - Disturbance : Factors affecting ambient temperature
  - Feedback : Measured temperature



Examples of Control Systems

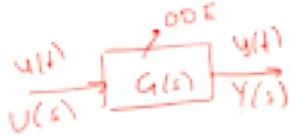
- **Human steering an automobile:**
  - Plant : Vehicle or automobile
  - Control system : Human control
  - Reference : Desired destination
  - Control Input : Steering mechanism
  - Output : Actual position
  - Disturbance : Traffic conditions



Transfer Function

- For an LTI system, transfer function is the ratio of the Laplace transform of the output to the Laplace transform of the input with the initial conditions being zero
- Mathematically, if $U(s)$ is the Laplace transform of the input function and $Y(s)$ is the Laplace transform of the output, the transfer function $G(s)$ is given by:

$$G(s) = \frac{Y(s)}{U(s)}$$



$$G(s) = \frac{Y(s)}{U(s)}$$

$$Y(s) = G(s)\, U(s)$$

Transfer Function as Impulse Response

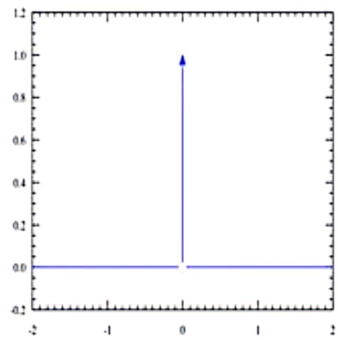
- Impulse signal ($\delta(t)$) is infinitesimally narrow and infinitely tall yet integrating to one
- It takes zero value everywhere except at $t = 0$

$$\int_{-\infty}^{\infty} \delta(t) = 1$$

- If input to the system is the unit impulse, then the output is called the impulse response i.e.,

$$u(t) = \delta(t) \Longrightarrow U(S) = 1 \Longrightarrow G(s) = Y(s)$$

- That means transfer function is the Laplace transform of the impulse response of an LTI system when the initial conditions are set to zero



Impulse function $\delta(t)$

$Y(s) = G(s) \cdot U(s)$

$Y(s) = G(s)$

Properties of Transfer Function

- ✓ Transfer function of a system is independent of the magnitude and nature of input

$U(s) \rightarrow \boxed{G(s)} \rightarrow y(t)$

- ✓ Using the transfer function, the response can be studied for various inputs to understand the nature of the system

$Y(s) = G(s) \cdot U(s)$

$y(t) = \mathcal{L}^{-1}(Y(s))$

- ✓ Transfer function does not provide any information concerning the physical structure of the system i.e., two different physical systems can have the same transfer function

E.g. MSD system : $G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2+Bs+K} = \frac{1}{s^2+s+1}$ $(M = B = K = 1)$

Series RLC circuit : $G(s) = \frac{V_0(s)}{V_i(s)} = \frac{1}{s^2LC+sRC+1} = \frac{1}{s^2+s+1}$ $(R = L = C = 1)$

Transfer Function : General Form

- General form of transfer function of a system:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0 s^m + b_1 S^{m-1} + \cdots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

$$= \frac{K'\big((s - z_1)(s - z_2) \ldots (s - z_m)\big)}{(s - p_1)(s - p_2) \ldots (s - p_n)}$$

  - n: Order of the system
  - K: System gain or Gain factor – A proportional value that relates the magnitude of the input to that of the output signal at steady state
  - $z_1, z_2, \ldots, z_m$: Zeros of the system
  - $p_1, p_2, \ldots, p_n$: Poles of the system
  - $n \geq m$ because the system becomes non-causal and is not physically realizable if $n < m$

Poles and Zeros

- **Poles:**
  - Roots of the denominator polynomial of the transfer function
  - Values of s at which the transfer function becomes unbounded

$$\lim_{s \to p_i} G(s) = \infty$$

$$\frac{s+1}{(s+2)(s+3)} = 0$$

$p_i = -2, -3 \uparrow \infty$

- **Zeros:**
  - Roots of the numerator polynomial of the transfer function
  - Values of s at which the transfer function vanishes

$$\lim_{s \to z_i} G(s) = 0$$

G

- Poles and zeros together with the system gain K characterise the input-output system dynamics

Gain, Poles and Zeros : Example

- Find the system gain, poles and zeros of the system with following transfer function: $\frac{6s+12}{s^3+3s^2+7s+5}$

6s +12 -

- $G(s) = \frac{6s+12}{s^3+3s^2+7s+5}$
- System gain: $K = \frac{12}{5}$
- Zeros: $s - 2 = 0 \Longrightarrow s = 2 \Rightarrow z_1 = 2$
- Poles: $s^3 + 3s^2 + 7s + 5 = 0 \Longrightarrow s = -1, -1 + 2j, -1 - 2j$

$$\Longrightarrow p_1 = -1, p_2 = -1 + 2j, p_3 = -1 - 2j$$

Note: Poles and zeros are purely real or appear in complex conjugates $(a \mp jb)$ because all the coefficients of transfer function are real

Block Diagram of a System

- It is a short hand pictorial representation of the system which depicts
  - Each functional component or sub-system and
  - Flow of signals from one sub-system to another
- Block diagram provides a simple representation of complex systems
- Block diagram enables calculating the overall system transfer function provided the transfer functions of each of the components or sub-systems are known

Components of Block Diagram

- Block diagrams have four components:
  1. **Blocks:** To represent the components or sub-systems

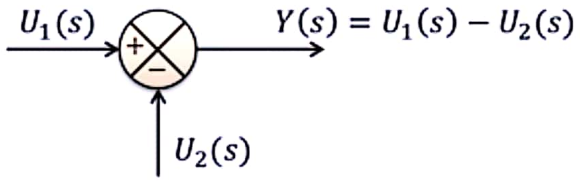
     $\boxed{G(s)}$ $G(s)$ is the transfer function of sub-system

  2. **Arrows:** To represent the direction of flow of signals

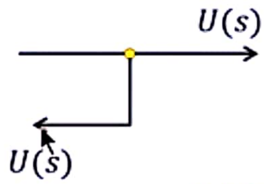
     $\xrightarrow{U(s)}$ $\xrightarrow{Y(s)}$

Components of Block Diagram

3. **Summing points:** To represent the summation of two or more signals

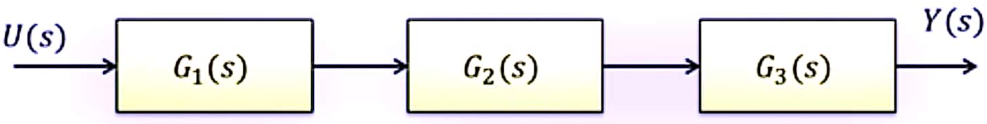


4. **Take-off points:** To represent the branching of a signal



Typical Block Diagram Forms

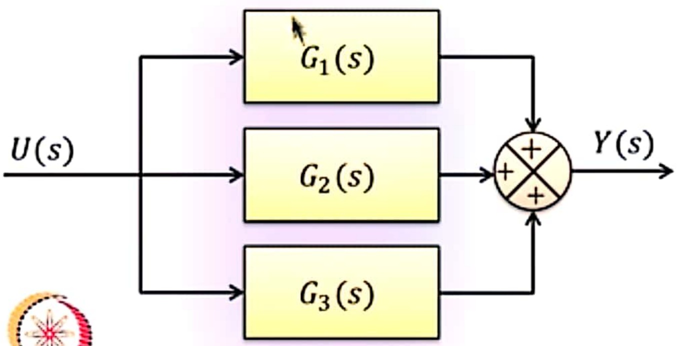
- **Cascaded Form / Series Form:**
  - Components or sub-systems of a system are connected in series each having its own transfer function
  - Overall transfer function is product of individual transfer functions



Transfer Function: $G(s) = \frac{Y(s)}{U(s)} = G_1(s)G_2(s)G_3(s)$

Typical Block Diagram Forms

- **Parallel Form:**
  - Components or sub-systems of a system are connected in parallel
  - Overall transfer function is sum of individual transfer functions



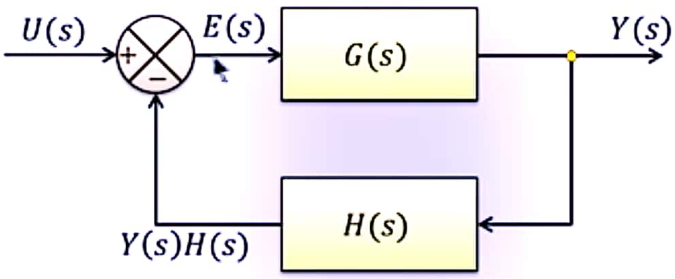
Transfer Function:

$$G(s) = \frac{Y(s)}{U(s)} = G_1(s) + G_2(s) + G_3(s)$$

Typical Block Diagram Forms

➢ **Feedback Form:**

– One component is present in the feedback loop of another component



Negative Feedback Loop

Transfer Function:

$$Y(s) = G(s)E(s)$$

$$Y(s) = G(s)[U(s) - Y(s)H(s)]$$

$$Y(s) = G(s)U(s) - G(s)H(s)Y(s)$$

$$Y(s)[1 + G(s)H(s)] = G(s)U(s)$$

$$\frac{Y(s)}{U(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

Review: Block Diagram of a System

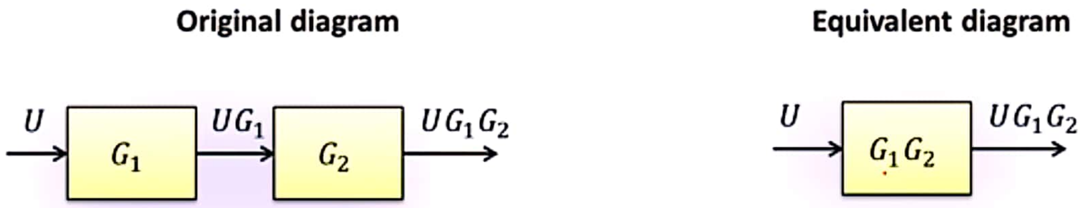
- It is a short hand pictorial representation of the system which depicts
  - Each functional component or sub-system and
  - Flow of signals from one sub-system to another
- Components of a block diagram:
  - Blocks to represent components
  - Arrows to indicate direction of signal flow
  - Summing points to show merging signals
  - Take off points to indicate branching of signals

Block Diagram Reduction

- Block diagram reduction refers to simplification of block diagrams of complex systems through certain rearrangements
- Simplification enables easy calculation of the overall transfer function of the system
- Simplification is done using certain rules called the 'rules of block diagram algebra'
- All these rules are derived by simple algebraic manipulations of the equations representing the blocks

Rules of Block Diagram Algebra

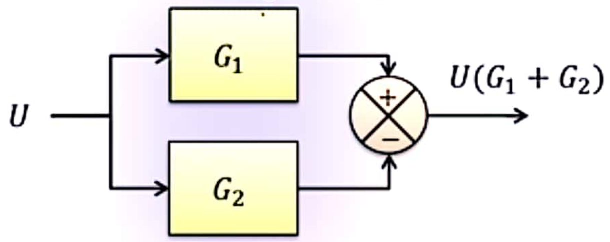
1. Combining blocks in cascade



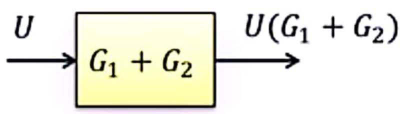
Rules of Block Diagram Algebra

2. Combining blocks in parallel

Original diagram



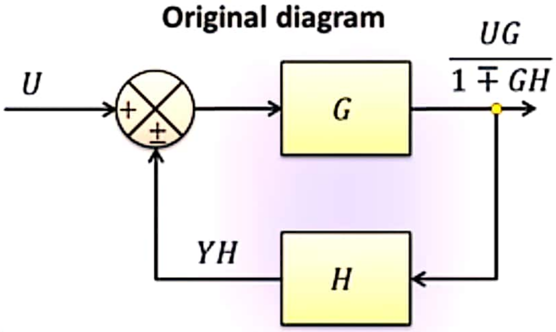
Equivalent diagram



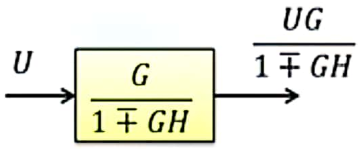
Rules of Block Diagram Algebra

3. Eliminating a feedback loop

Original diagram

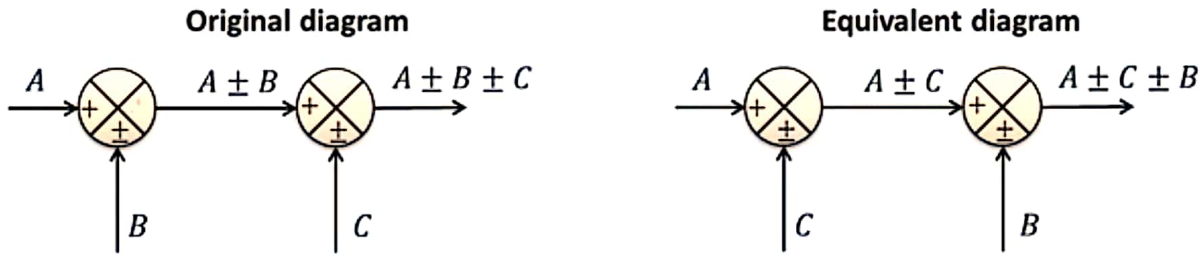


Equivalent diagram



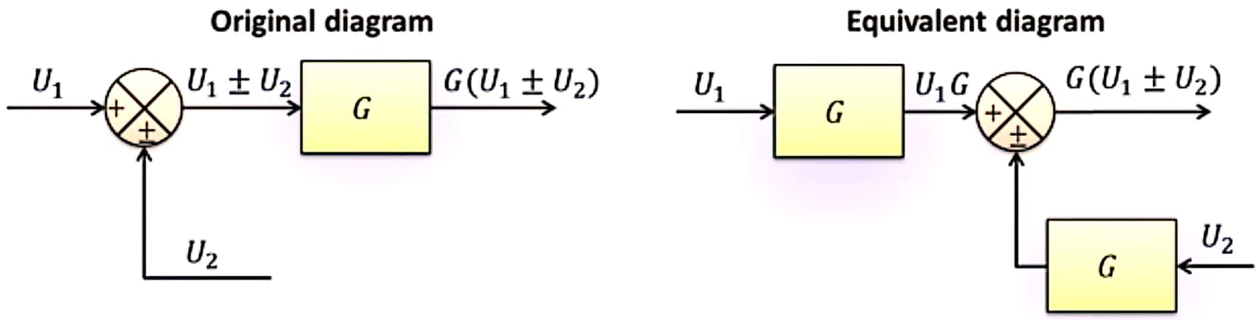
Rules of Block Diagram Algebra

4. Interchanging the summing point



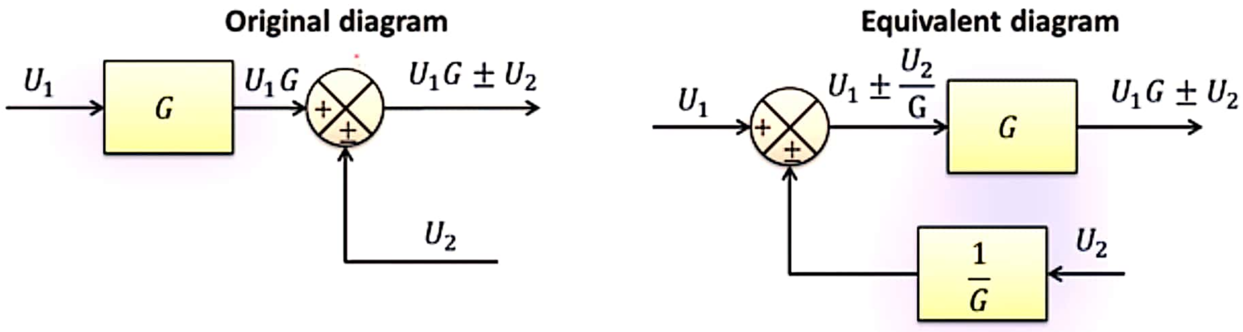
Rules of Block Diagram Algebra

5. Moving a summing point after a block



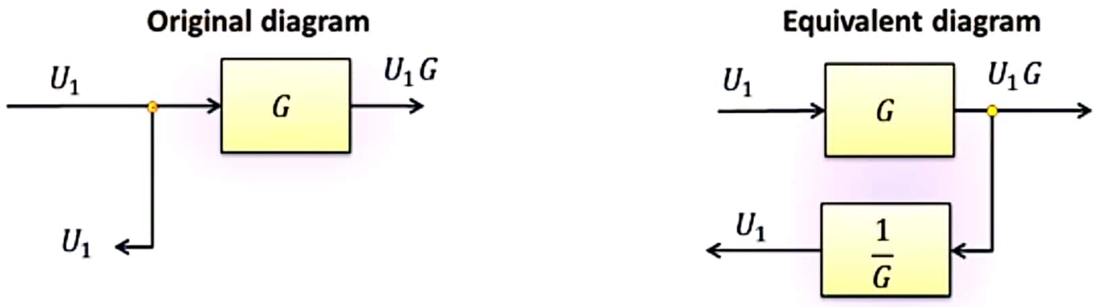
Rules of Block Diagram Algebra

6. Moving a summing point ahead of a block



Rules of Block Diagram Algebra

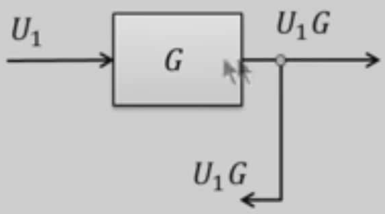
7. Moving a take-off point after a block



Rules of Block Diagram Algebra

8. **Moving a take-off point ahead of a block**

Original diagram



Equivalent diagram