ENGG. MATH. - II

(Code : BST-201)

Full Marks : 80   *Time* : 3 hours

Answer any **five** questions including **Q. Nos. 1 & 2**

Figures in the right-hand margin indicate marks

1. Answer *all* questions : 2×10

   (*a*) Evaluate

   $$\lim_{x \to 1} \frac{x^2 - 2x + 1}{x^2 - x}.$$

   (*b*) For what value of K

   $$f(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a}, & \text{if } x \neq a \\ K, & \text{if } x = a \end{cases}$$

   is continuous at $x = a$.

   (*c*) Evaluate

   $$\int_0^1 \frac{1}{1 + x^2}\, dx$$

   (*d*) Find the equation of the sphere with centre $(2, -1, 4)$ and radius 4.

   (*e*) Differentiate $\log(\sin x)$ w.r.t. $\tan x$.

   (*f*) Find order and degree of the differential equation

   $$\frac{d^2 y}{dx^2} = \left\{ 2 + \left(\frac{dy}{dx} \right)^3 \right\}^{1/2}$$

   (*g*) If $z = \tan^{-1}\left(\dfrac{x}{y} \right)$, find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

   (*h*) Find y_1 and y_2 if $y = \log(\cos x)$.

   (*i*) Find the direction cosines of the line joining the points $(1, 2, 3)$ and $(3, 0, 2)$.

   (*j*) Evaluate

   $$\int \frac{e^{2x} + 1}{e^x}\, dx.$$

2. Answer any *six* questions : 5×6

   (*a*) Evaluate

   $$\lim_{x \to 0} \left(\frac{x \tan x}{1 - \cos x} \right).$$

(b) Examine the continuity of the function defined by

$$f(x) = \begin{cases} \dfrac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \quad \text{at } x = 0$$

(c) If $y = \sin^{-1}x$, show that

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0.$$

(d) If $y = \tan^{-1}\sqrt{\dfrac{1 - \cos x}{1 + \cos x}}$, then find $\dfrac{dy}{dx}$.

(e) Determine the maximum and minimum value of $f(x) = x^3 - 6x^2 + 9x + 7$.

(f) Evaluate

$$\int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}}\, dx.$$

(g) Find the area bounded by the curve

$$y^2 = x, \quad x = 0, \quad y = 1.$$

(h) Find the equation of the plane which passes through the point $(4, -2, 1)$ and is perpendicular to the line whose direction ratios are $5, 2, -1$.

3. Differentiate :

$$\sin^2\left\{\cot^{-1}\sqrt{\frac{1 + x}{1 - x}}\right\}$$

10

4. Evaluate

$$\int \frac{4x^2 - x + 3}{(x^2 + 1)(x - 1)}\, dx$$

10

5. Find the equation of the sphere which passes through the points $(0, 0, 0)$, $(-a, b, c)$, $(a, -b, c)$, $(a, b, -c)$.

10

6. Solve :

$$4\frac{dy}{dx} + 8y = 5e^{-3x}$$

10

7. If $V = (x^2 + y^2 + z^2)^{1/2}$, show that

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0.$$

10

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