ENGINEERING MATHEMATICS-I
(Code : BST-103)

Full Marks : 80   *Time* : 3 hours

Answer any **five** questions including **Q. Nos. 1 & 2**

Figures in the right-hand margin indicate marks

1. Answer *all* questions : 2×10

   (a) Find the value of $(-i)^{4m+5}$.

   (b) Evaluate $\begin{vmatrix} w^6 & w^4 \\ -w^6 & w^5 \end{vmatrix}$ where $w^3 = 1$.

   (c) Find the value of $\cos\left(\sin^{-1}\frac{1}{4} + \cos^{-1}\frac{1}{4}\right)$.

   (d) If $\sin\alpha = \frac{1}{2}$ and $\sin\beta = \frac{1}{3}$, find the value of $\sin(\alpha + \beta)$.

   (e) Find the equation of the line whose x-intercept is 3 and y-intercept is 4.

   (f) Determine the centre and radius of the circle $x^2 + y^2 - 6x + 4y - 36 = 0$.

   (g) Find the adjoint of the matrix :

   $$\begin{bmatrix} i & -i \\ i & i \end{bmatrix}.$$

   (h) Find the value of 'p' so that the vectors $2\hat{i} + \hat{j} - \hat{k}$ is perpendicular to the vector $\hat{i} - \hat{j} + p\hat{k}$.

   (i) Find the number of terms in the expansion of $\left(x^2 - 2 + \frac{1}{x^2}\right)^7$.

   (j) If $\sin A = \sin B$ and $b = \frac{1}{2}$, then find the value of 'a'.

2. Answer any *six* questions : 5×6

   (a) Find the square root of $-5 + 12\sqrt{-1}$.

   (b) Find the adjoint of the matrix :

   $$\begin{bmatrix} 1 & 1 & -1 \\ 2 & -1 & 2 \\ 1 & 3 & -2 \end{bmatrix}.$$

   (c) If $A + B + C = \pi$, prove that

   $$\sin 2A + \sin 2B + \sin 2C = 4\sin A \cdot \sin B \cdot \sin C$$

   (d) Prove that $\tan 37\frac{1}{2}^{\circ} = \sqrt{6} + \sqrt{3} - \sqrt{2} - 2$.

(*e*) Obtain the equation of a circle passing through the points (0, 0), (6, 0) and (0, 8).

(*f*) Obtain the equation of the line passing through the point (– 2, 3) and perpendicular to the line $3x + 4y – 11 = 0$.

(*g*) Determine the area of a parallelogram whose diagonals are determined by the vectors $\vec{a} = 3\hat{i} + \hat{j} – 2\hat{k}$ and $\vec{b} = \hat{i} – 3\hat{j} + 4\hat{k}$.

(*h*) In any triangle, prove that

$$b \cos B + c \cos C = a \cos (B – C).$$

3. Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3.$ 10

4. Solve the following equation by matrix method :   10

$$x + y + z = 4$$
$$2x – y + 3z = 1$$
$$3x + 2y – z = 1$$

5. Resolve into partial fractions :   10

$$\frac{x}{(x-1)(x^2+1)}.$$

6. Find the equation of the line passing through the intersection of $2x – y – 1 = 0$ and $3x – 4y + 6 = 0$ and parallel to the line $x + y – 2 = 0$.   10

7. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, show that

$$x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz.$$ 10

––––––––––––