I – Sem/COMMON/2018(W)(New)(BP)

ENGINEERING MATHEMATIC - I

(Code : BST-103)

Full Marks : 80

Time : 3 hours

Answer any **five** questions including **Q. Nos. 1** and **2**

Figures in the right-hand margin indicate marks

1. Answer *all* questions : 2×10

(*i*) If w is imaginary cube-roots of unity then find the value of $w^2 + w^3 + w^4$.

(*ii*) Find the value of

$$\frac{\cos 15^\circ + \sin 15^\circ}{\cos 15^\circ - \sin 15^\circ}$$

(*iii*) Find the radius of the circle

$$x^2 + y^2 - 2x - 2y + z = 0$$

(*iv*) If the matrix $A = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$, prove that $A^2 = A$.

(*v*) Find the value of

$$\tan\left[\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}\right]$$

(*vi*) If the slope and X-intercept of the line $3x - y + k = 0$ are equal, then find the value of k.

(*vii*) Find the equation of the circle whose centre is at (0, 0) and the circle passes through the point (2, 3).

(*viii*) Find the unit vector in the direction of the vector $\hat{j} - \hat{k}$

(*ix*) Compute :

$${}^7C_3 + {}^6C_4 + {}^6C_3$$

(*Turn Over*)

(x) Solve by Cramer's rule

$$2x - y = 3,\ x + 2y = 4$$

2. Answer any *six* questions : 5×6

(*i*) Find the square root of

$$-8 + \sqrt{-1}$$

(*ii*) Prove that

$$\cot^{-1} 9 + \operatorname{cosec}^{-1} \frac{\sqrt{41}}{4} = \frac{\pi}{4}$$

(*iii*) Prove that

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

(*iv*) If $A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$, show that $A^2 = A$.

(*v*) Obtain the equation of straight line bisecting the line segment (3, –4) and (1, 2) at right angles.

(*vi*) Find the middle term in the expansion of $\left(\frac{a}{b} + \frac{b}{a}\right)^{10}$.

(*vii*) Find the scalar and vector projection of $\vec{a}$ on $\vec{b}$ where $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$.

3. Split into partial fraction

$$\frac{4x^2 - x + 3}{(x^2 + 1)(x - 1)}$$ 10

4. Solve by matrix method 10

$$\begin{aligned} x + y + z &= 4 \\ 2x + 5y - 2z &= 3 \\ x + 7y - 7z &= 5 \end{aligned}$$

5. Obtain the equation of the circle passing through the points $(-3, 1), (5, -3)$ and $(-3, 4)$ also find the co-ordinates of centre and radius of the circle. 10

6. In any triangle ABC, prove that

$$\sum \frac{a^2 \sin(B-C)}{\sin(B+C)} = 0$$ 10

7. If $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}, \vec{b} = -\hat{i} + 2\hat{j} - 4\hat{k}, \vec{c} = \hat{i} + \hat{j} + \hat{k}$, find $(\vec{a} \times \vec{b}) \cdot (\vec{a} \times \vec{c})$. 10